From Euler Diagrams to Aristotelian Diagrams*

Lorenz Demey[1] [0000−0002−0176−1958] and Hans Smessaert[2] [0000−0002−8186−0170]

[1] Center for Logic and Philosophy of Science, KU Leuven
Kardinaal Mercierplein 2, B-3000 Leuven, Belgium
lorenz.demey@kuleuven.be

[2] Department of Linguistics, KU Leuven
Blijde-Inkomststraat 21, B-3000 Leuven, Belgium
hans.smessaert@kuleuven.be

Abstract. Euler and Aristotelian diagrams are both among the most well-studied kinds of logical diagrams today. Despite their central status, very little research has been done on relating these two types of diagrams. This is probably due to the fact that Euler diagrams typically visualize relations between sets, whereas Aristotelian diagrams typically visualize relations between propositions. However, recent work has shown that Aristotelian diagrams can also perfectly be understood as visualizing relations between sets, and hence it becomes natural to ask whether there is any kind of systematic relation between Euler and Aristotelian diagrams. In this paper we provide an affirmative answer: we show that every Euler diagram for two non-trivial sets gives rise to a well-defined Aristotelian diagram. Furthermore, depending on the specific relation between the two sets visualized by the Euler diagram, the resulting Aristotelian diagram will also be fundamentally different. We will also link this with well-known notions from logical geometry, such as the information ordering on the seven logical relations between non-trivial sets, and the notion of Boolean complexity of Aristotelian diagrams.

Keywords: Euler diagram · Aristotelian diagram · square of opposition · logical geometry · information ordering · Boolean complexity.

1 Introduction

Euler diagrams are among the most well-studied kinds logical diagrams today [1, 20, 24, 26, 27]. They have a rich history, which obviously includes the work of Leonhard Euler in the eighteenth century, but also goes back much further, at least to medieval manuscripts from the eleventh century [11, 16–18]. Similarly, Aristotelian diagrams (such as the square of opposition) are also studied intensively today, especially in the burgeoning research program of logical geometry [3, 9, 10, 25], and they, too, can boast a long and well-documented history [13, 21, 23]. The history of both types of diagrams is further described in [22].

* The first author holds a Research Professorship (BOFZAP) from KU Leuven. This research was funded through the KU Leuven research project 'BITSHARE: Bitstring Semantics for Human and Artificial Reasoning' (3H190254, 2019 – 2023).

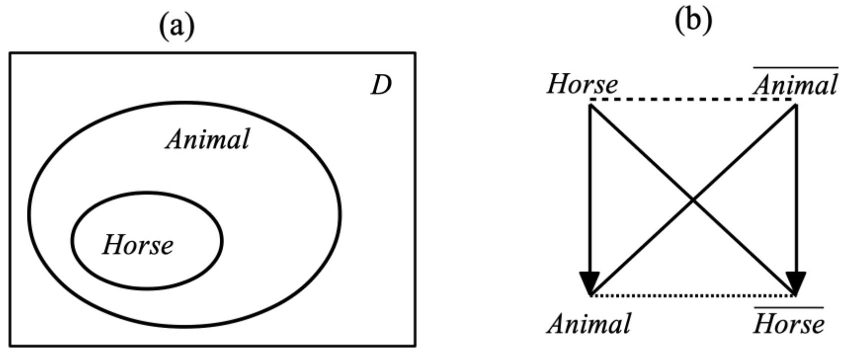

Fig. 1. (a) Euler diagram for $A \subset B$; (b) the corresponding square of opposition. (Solid, dashed and dotted lines respectively stand for contradiction, contrariety and subcontrariety; arrows stand for subalternation.)

Despite their central status, very little research has been done thus far on relating Euler and Aristotelian diagrams.[3] One explanation for this lacuna might be that Euler diagrams typically visualize a relation between *sets/terms*, whereas Aristotelian diagrams typically visualize relations between *propositions/sentences*. However, recent work has shown that the mathematical background structure required for obtaining a well-defined Aristotelian diagram is that of a Boolean algebra, and it does not matter whether this is an algebra consisting of propositions or of terms (or yet some other notion) [4]. Consequently, it becomes a mathematically well-defined and conceptually natural question to ask whether there is any kind of systematic connection between Euler and Aristotelian diagrams. Our goal in this paper is to provide an affirmative answer to this question. In particular, we will show how each Euler diagram for two (non-trivial) sets gives rise to a well-defined Aristotelian diagram. Furthermore, depending on the specific relation between the two sets visualized by the Euler diagram, the resulting Aristotelian diagram will also be fundamentally different.

The paper is organized as follows. Section 2 presents a motivating example and describes some of the necessary theoretical background. Section 3 contains the main results of this paper, and shows how each two-set Euler diagram gives rise to an Aristotelian diagram. Section 4 presents some further discussion of these results, and mentions some questions for future research.

2 Motivating Example and Theoretical Background

Consider the Euler diagram shown in Fig. 1(a). What does this diagram represent or visualize? The standard answer is that the diagram visualizes two sets, *Horse* and *Animal* (which exist inside some domain of discourse D), and the relation

[3] A notable exception, albeit in a very different direction than the one we will take in this paper, is the work of Bernhard [2].

Horse $\subset$ *Animal*, i.e., *Horse* is a strict subset of *Animal*. Relative to a Boolean algebra of sets, this means that there exists a subalternation from *Horse* to *Animal*.[4] However, according to a less standard answer, the diagram in Fig. 1(a) shows *more* than what has just been mentioned. First of all, by showing the set *Horse* inside the domain of discourse D, it also shows, if only implicitly, the complement set $\overline{Horse} = D \backslash Horse$. The same goes for the set *Animal* and its complement $\overline{Animal} = D \backslash Animal$. Secondly, the Euler diagram also shows, again only implicitly, five more relationships that these two new complement sets enter into:

- $Horse \cap \overline{Horse} = \emptyset$ and $Horse \cup \overline{Horse} = D$,
  i.e., *Horse* and $\overline{Horse}$ are contradictory to each other,
- $Animal \cap \overline{Animal} = \emptyset$ and $Animal \cup \overline{Animal} = D$,
  i.e., *Animal* and $\overline{Animal}$ are contradictory to each other,
- $Horse \cap \overline{Animal} = \emptyset$ and $Horse \cup \overline{Animal} \neq D$,
  i.e., *Horse* and $\overline{Animal}$ are contrary to each other,
- $Animal \cap \overline{Horse} \neq \emptyset$ and $Animal \cup \overline{Horse} = D$,
  i.e., *Animal* and $\overline{Horse}$ are subcontrary to each other,
- $\overline{Animal} \subset \overline{Horse}$, i.e., there is a subalternation from $\overline{Animal}$ to $\overline{Horse}$.

Taken together, these four sets and the six relations holding among them can be visualized by means of a square of opposition, as shown in Fig. 1(b).[5] This Aristotelian diagram thus contains exactly the same information (i.e., the same sets and the same relations among them) as the Euler diagram in Fig. 1(a). The only difference between both diagrams is that the Euler diagram strongly emphasizes two sets, viz. *Horse* and *Animal*, and one relation, viz. the subalternation from *Horse* to *Animal*, while strongly 'downplaying' the two other sets and the five other relations. By contrast, the Aristotelian diagram attributes

[4] Given an arbitrary Boolean algebra $\mathbb{B}$, we say that x and y are contradictory iff $x \wedge_{\mathbb{B}} y = \bot_{\mathbb{B}}$ and $x \vee_{\mathbb{B}} y = \top_{\mathbb{B}}$, that they are contrary iff $x \wedge_{\mathbb{B}} y = \bot_{\mathbb{B}}$ and $x \vee_{\mathbb{B}} y \neq \top_{\mathbb{B}}$, that they are subcontrary iff $x \wedge_{\mathbb{B}} y \neq \bot_{\mathbb{B}}$ and $x \vee_{\mathbb{B}} y = \top_{\mathbb{B}}$, and that they are in subalternation iff $x <_{\mathbb{B}} y$. If $\mathbb{B}$ happens to consist of subsets of some given set D, this means that X and Y are contradictory iff $X \cap Y = \emptyset$ and $X \cup Y = D$, that they are contrary iff $X \cap Y = \emptyset$ and $X \cup Y \neq D$, that they are subcontrary iff $X \cap Y \neq \emptyset$ and $X \cup Y = D$, and that they are in subalternation iff $X \subset Y$. See [4, Section 2] for further explanation and motivation.

[5] This square might look a bit strange, since it contains sets rather than propositions. However, we emphasize once again that Aristotelian relations (and thus also diagrams) can be defined relative to arbitrary Boolean algebras, regardless of whether these algebras consist of propositions, sets, or something else. The square of opposition in Fig. 1(b) is thus perfectly well-defined, just like any other, more ordinary-looking square of opposition that contains propositions rather than sets.

equal status to all four sets and all six relations alike.[6] Some of these ideas were already mentioned in passing in a recent, more historically oriented paper:[7]

> one can view the original Euler diagram in Fig. [1(a)] as a visual representation of both proper inclusion relations—albeit, perhaps, with different degrees of visual perspicuity. More generally, from this alternative perspective, the single Euler diagram in Fig. [1(a)] at once visualizes six relations among *Horse*, *Animal*, $D\backslash$*Horse* and $D\backslash$*Animal*, all six of which are also visualized by the classical square of opposition in Fig. [1(b)].
>
> [5, p. 192, references to figures updated to the present paper]

In this paper, we will investigate these ideas more systematically. More specifically, we will show that this kind of tranformation not only works for Euler diagrams representing a subalternation relation, but also for Euler diagrams depicting any other kind of relation between two sets. In [25] it is shown that every pair of non-trivial[8] sets X and Y (within a domain of discourse D) stands in exactly one of the following seven relations:[9]

1. contradiction (CD): $X \cap Y = \emptyset$ and $X \cup Y = D$,
2. contrariety (C): $X \cap Y = \emptyset$ and $X \cup Y \neq D$,
3. subcontrariety (SC): $X \cap Y \neq \emptyset$ and $X \cup Y = D$,
4. bi-implication (BI): $X \subseteq Y$ and $X \supseteq Y$, i.e. $X = Y$,
5. left-implication (LI): $X \subseteq Y$ and $X \not\supseteq Y$, i.e. $X \subset Y$,
6. right-implication (RI): $X \not\subseteq Y$ and $X \supseteq Y$, i.e. $X \supset Y$,
7. unconnectedness (UN): $X \cap Y \neq \emptyset$ and $X \cup Y \neq D$ and $X \not\subseteq Y$ and $X \not\supseteq Y$.

The first three relations are sometimes called *opposition relations*, while the next three are the *implication relations*. Note that left-implication corresponds to the ordinary Aristotelian relation of subalternation. Unconnectedness can be viewed as the absence of any other relation: X and Y are unconnected iff they do not stand in any of the other relations. These seven relations constitute a refinement of the five so-called 'Gergonne relations', which are perhaps more widely known

[6] In earlier work [7, 8], we have argued that the idea that a square of opposition attributes *exactly* the same status to all six relations should be somewhat nuanced. For example, based on principles like center/periphery or on considerations regarding distance, one could argue that contradiction (on the two diagonals, in the center of the square) is visualized more prominently than the other relations (on the edges, at the periphery of the square). However, these subtle differences in an Aristotelian diagram completely vanish in comparison to the more drastic differences in emphasis that occur in Euler diagrams — e.g. the explicit subalternation from *Horse* to *Animal* versus the more implicit subcontrariety between *Animal* and $\overline{Horse}$ in Fig. 1(a).

[7] The theoretical core of [5] consists of its Sections 3, 4 and 5. The present paper can be viewed as building upon Section 3, while [19] elaborates on Sections 4 and 5.

[8] Given a domain of discourse D, a set X is said to be *non-trivial* iff $\emptyset \neq X \neq D$.

[9] These seven relations could also be defined for arbitrary Boolean algebras instead of just for sets. However, for the purposes of this paper this will not be necessary.

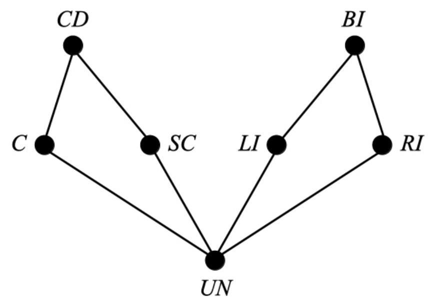

Fig. 2. Information ordering on the seven relations between two non-trivial sets [25].

[12, 14]. The Gergonne relations $X = Y$, $X \subset Y$ and $X \supset Y$ straightforwardly correspond to *BI*, *LI* and *RI*, respectively; furthermore, the Gergonne relation $X \cap Y = \emptyset$ corresponds to *CD* $\cup$ *C*, and $X \cap Y \neq \emptyset$ corresponds to *SC* $\cup$ *UN*. Finally, these seven relations are ordered according to their information levels [25]: it can be shown that contradiction and bi-implication are the most informative relations, unconnectedness is the least informative, and the four other relations' information levels are in between. This information ordering is shown in Fig. 2.

3 The Seven Euler Diagrams for Two Sets and their Corresponding Aristotelian Diagrams

We will now consider Euler diagrams for each of the seven possible relations between two (non-trivial) sets, and investigate what kind of Aristotelian diagram they give rise to. We start with the implication relation of *left-implication*, which boils down to re-examining the motivating example from the previous section. The Euler diagram in Fig. 3(a) shows a left-implication (i.e., subalternation) from A to B. In order to highlight the six relations that are shown by this diagram, we will use thick black and grey ellipses for resp. A and B, and thick black and grey dashed lines, together with a thickened rectangle for the domain of discourse, for their complements, resp. $\overline{A}$ and $\overline{B}$.[10] Using this highlighting convention, Fig. 3(c) and (d) show the very same Euler diagram as in (a), but now highlighting the subalternations from A to B and from $\overline{B}$ to $\overline{A}$, respectively. Similarly, Fig. 3(e) highlights the contrariety between A and $\overline{B}$, while Fig. 3(f) highlights the subcontrariety between $\overline{A}$ and B. Finally, Fig. 3(g) and (h) highlight the contradictions between A and $\overline{A}$ and between B and $\overline{B}$, respectively. We emphasize once more that Figs. 3(c–h) should not be viewed as six separate Euler diagrams, but rather as six ways of looking at one and the

[10] The overall idea is thus that a set corresponds to the region delimited by a thick solid line (either an ellipse or the outer rectangle), subtracting (if necessary/applicable) the region inside the thick dashed line.

same Euler diagram, viz. the one in Fig. 3(a). Needless to say, some of these six relations — e.g. the subalternation from A to B highlighted in Fig. 3(c) — are far easier to process than some of the others — e.g. the subcontrariety between $\overline{A}$ and B shown in Fig. 3(f). Taken together, these six relations (all of which are Aristotelian) constitute the classical square of opposition shown in Fig. 3(b).

Secondly, we consider the implication relation of *right-implication*. The Euler diagram in Fig. 4(a) shows a right-implication from A to B, i.e. a subalternation from B to A. Using the same convention as before, Fig. 4(c) and (d) show the very same Euler diagram as in (a), but now highlighting the subalternations from B to A and from $\overline{A}$ to $\overline{B}$, respectively. Similarly, Fig. 4(e) highlights the subcontrariety between A and $\overline{B}$, while Fig. 4(f) highlights the contrariety between $\overline{A}$ and B. Finally, Fig. 4(g) and (h) highlight the contradictions between A and $\overline{A}$ and between B and $\overline{B}$, respectively. Taken together, these six relations (all of which are Aristotelian) constitute the classical square of opposition shown in Fig. 4(b).

Thirdly, we switch over to the opposition relations, and consider the relation of *contrariety*. The Euler diagram in Fig. 5(a) shows a contrariety between A and B. Fig. 5(c) highlights the contrariety between A and B, while Fig. 5(d) highlights the subcontrariety between $\overline{A}$ and $\overline{B}$. Similarly, Fig. 5(e) and (f) highlight the subalternations from A to $\overline{B}$ and from B to $\overline{A}$, respectively. Finally, Fig. 5(g) and (h) highlight the contradictions between A and $\overline{A}$ and between B and $\overline{B}$, respectively. Taken together, these six relations (all of which are Aristotelian) constitute the classical square of opposition shown in Fig. 5(b).

Fourthly, we consider the opposition relation of *subcontrariety*. The Euler diagram in Fig. 6(a) shows a subcontrariety between A and B. Fig. 6(c) highlights the subcontrariety between A and B, while Fig. 6(d) highlights the contrariety between $\overline{A}$ and $\overline{B}$. Similarly, Fig. 6(e) and (f) highlight the subalternations from $\overline{B}$ to A and from $\overline{A}$ to B, respectively. Finally, Fig. 6(g) and (h) highlight the contradictions between A and $\overline{A}$ and between B and $\overline{B}$, respectively. Taken together, these six relations (all of which are Aristotelian) constitute the classical square of opposition shown in Fig. 6(b).

Fifthly, we switch back to the implication relations, and consider the relation of *bi-implication*. The Euler diagram in Fig. 7(a) shows a bi-implication between A and B. Fig. 7(c) and (d) highlight the bi-implications between A and B and between $\overline{A}$ and $\overline{B}$, respectively. Similarly, Fig. 7(e) and (f) highlight the contradictions between A and $\overline{B}$ and between $\overline{A}$ and B, respectively. Finally, Fig. 7(g) and (h) highlight the contradictions between A and $\overline{A}$ and between B and $\overline{B}$, respectively. Taken together, these six relations constitute the pair of contradictories (PCD) shown in Fig. 7(b). This PCD contains two identical sets at both of its vertices, which correspond to the bi-implications A/B and $\overline{A}/\overline{B}$ (which are themselves not Aristotelian). The single solid line corresponds to *four* contradiction relations $A/\overline{A}$, $A/\overline{B}$, $B/\overline{A}$ and $B/\overline{B}$ (which are Aristotelian).

Sixthly, we switch back one more time to the opposition relations, and consider the relation of *contradiction*. The Euler diagram in Fig. 8(a) shows a contradiction between A and B. Fig. 8(c) and (d) highlight the contradictions between A and B and between $\overline{A}$ and $\overline{B}$, respectively. Similarly, Fig. 8(e) and (f) highlight

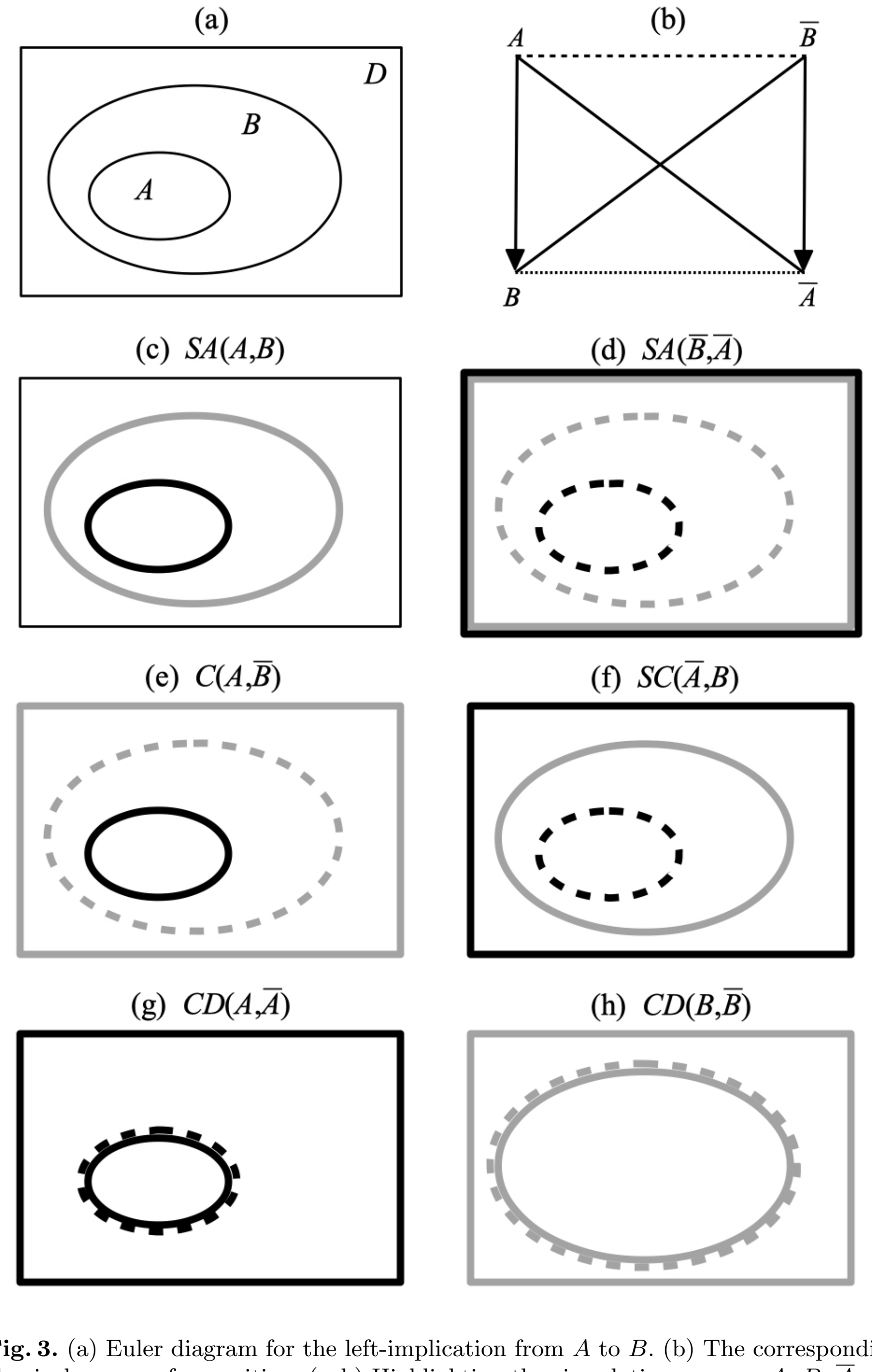

Fig. 3. (a) Euler diagram for the left-implication from A to B. (b) The corresponding classical square of opposition. (c–h) Highlighting the six relations among A, B, $\overline{A}$ and $\overline{B}$ in the Euler diagram (note: all six of them are Aristotelian relations).

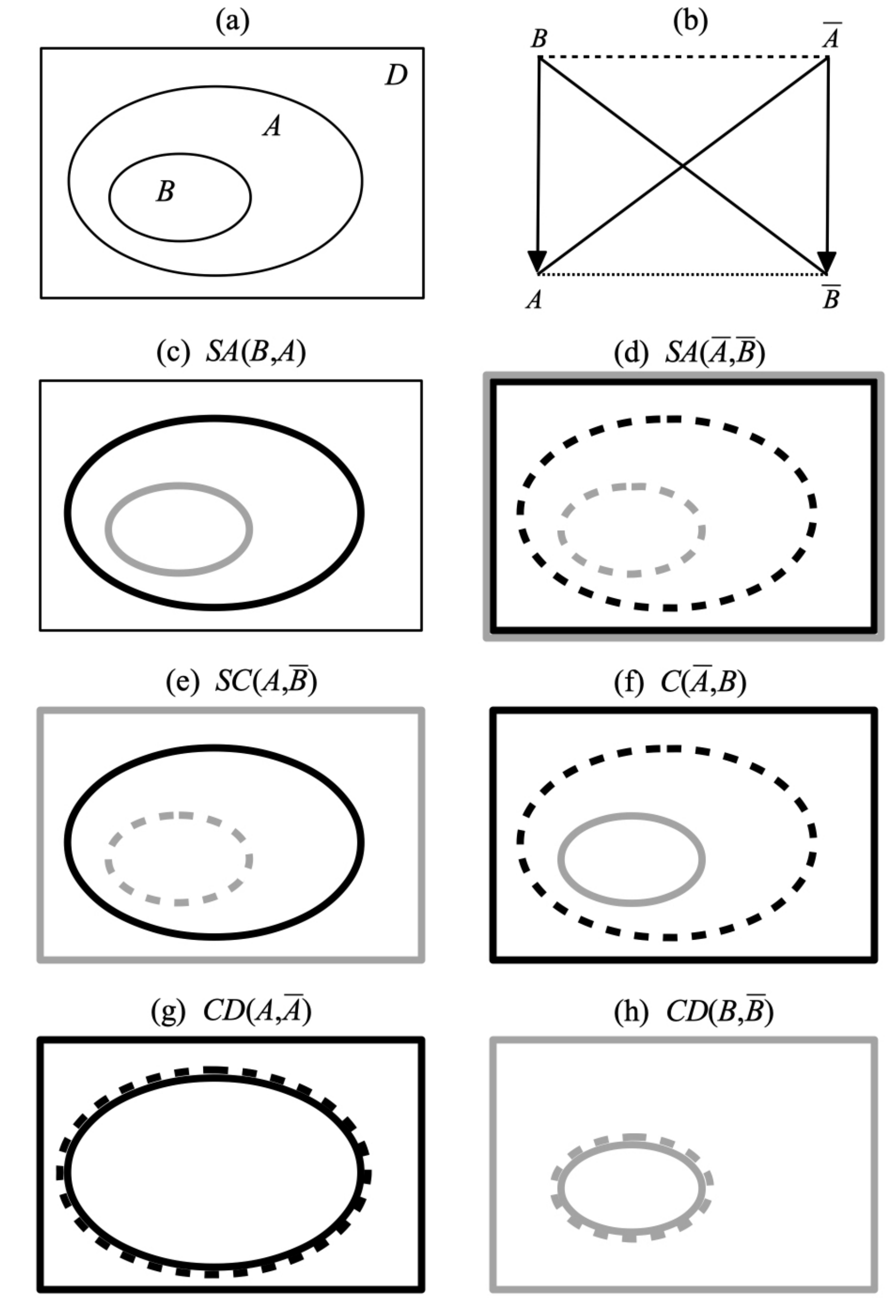

Fig. 4. (a) Euler diagram for the right-implication from A to B. (b) The corresponding classical square of opposition. (c–h) Highlighting the six relations among A, B, $\overline{A}$ and $\overline{B}$ in the Euler diagram (note: all six of them are Aristotelian relations).

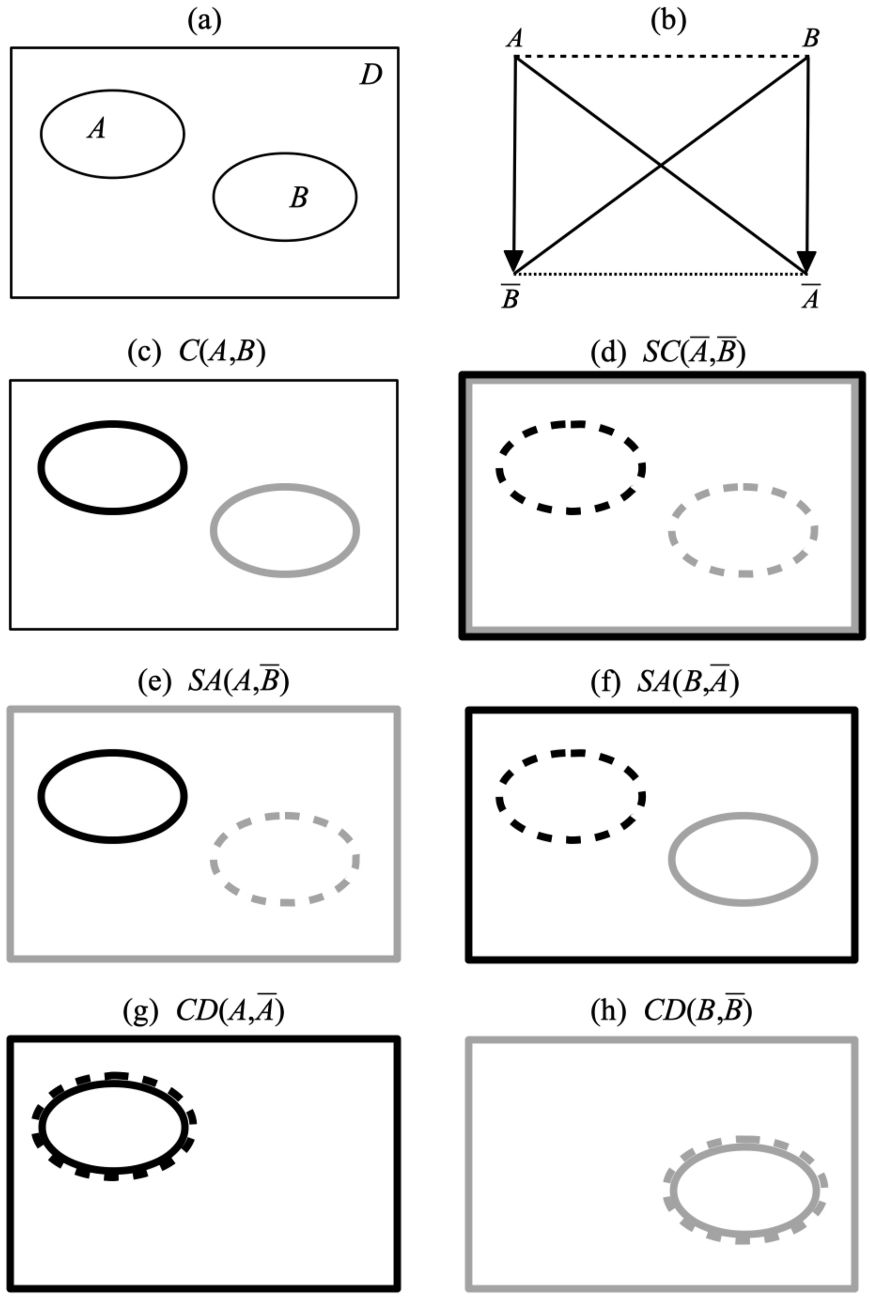

Fig. 5. (a) Euler diagram for the contrariety between A and B. (b) The corresponding classical square of opposition. (c–h) Highlighting the six relations among A, B, $\overline{A}$ and $\overline{B}$ in the Euler diagram (note: all six of them are Aristotelian relations).

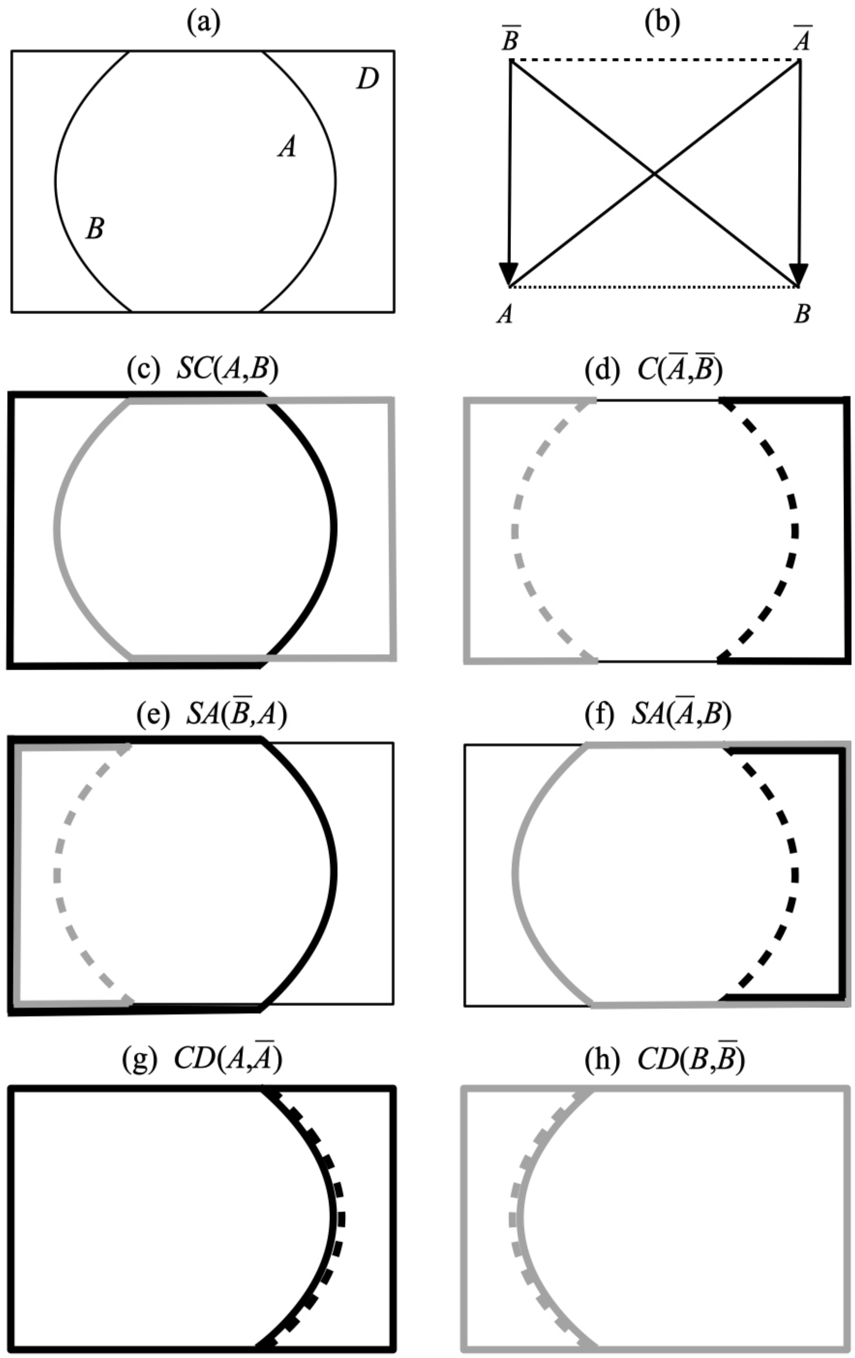

Fig. 6. (a) Euler diagram for the subcontrariety between A and B. (b) The corresponding classical square of opposition. (c–h) Highlighting the six relations among A, B, $\overline{A}$ and $\overline{B}$ in the Euler diagram (note: all six of them are Aristotelian relations).

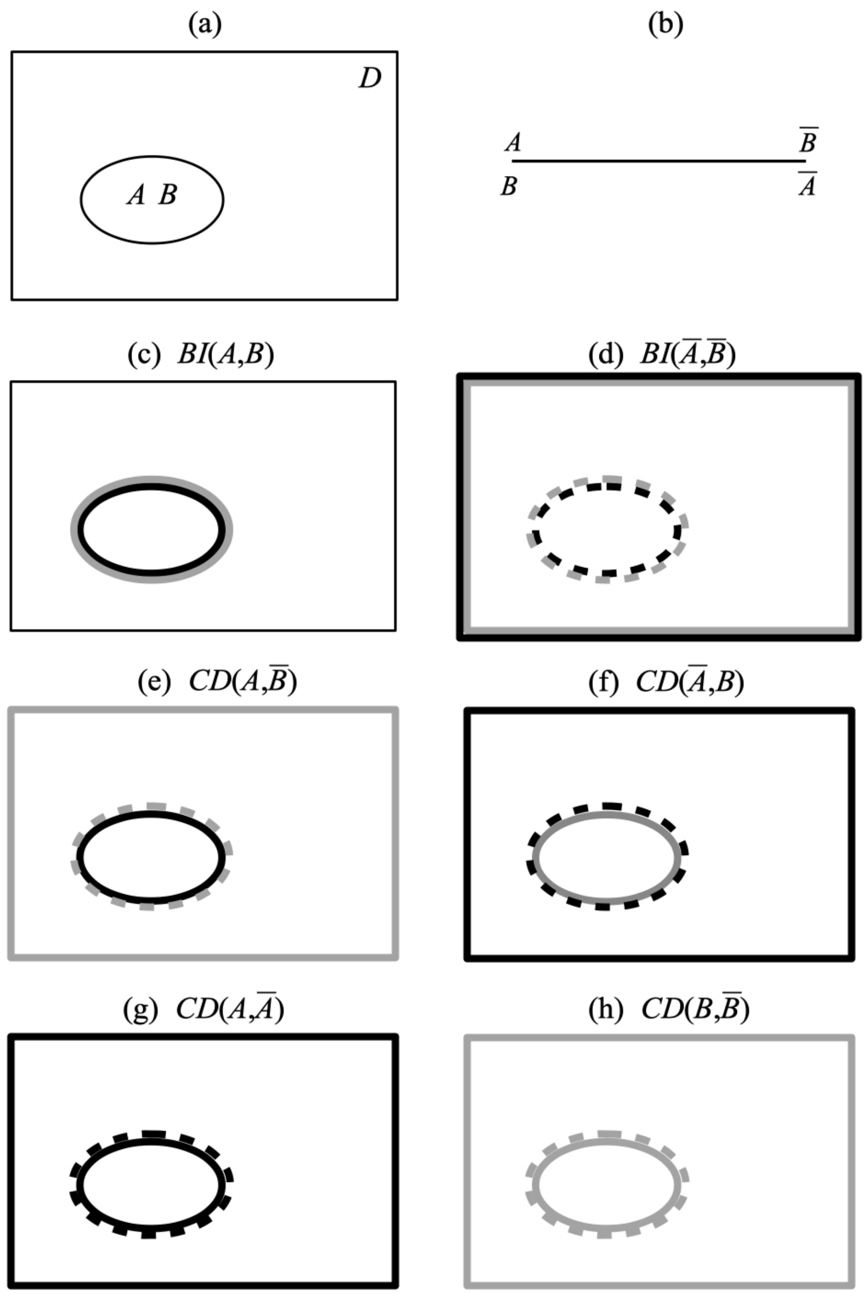

Fig. 7. (a) Euler diagram for the bi-implication between A and B. (b) The corresponding PCD. (c–h) Highlighting the six relations among A, B, $\overline{A}$ and $\overline{B}$ in the Euler diagram (note: the four CD are Aristotelian relations, but the two BI are not).

the bi-implications between A and $\overline{B}$ and between $\overline{A}$ and $\overline{B}$, respectively. Finally, Fig. 8(g) and (h) highlight the contradictions between A and $\overline{A}$ and between B and $\overline{B}$, respectively. Taken together, these six relations (four Aristotelian *CD* and two non-Aristotelian *BI*) constitute another PCD, as shown in Fig. 8(b).

Finally, we consider the relation of *unconnectedness*, which is neither a genuine opposition relation nor a genuine implication relation. The Euler diagram in Fig. 9(a) shows an unconnectedness between A and B. Fig. 9(c) and (d) highlight the unconnectedness between A and B and between $\overline{A}$ and $\overline{B}$, respectively. Similarly, Fig. 9(e) and (f) highlight the unconnectedness between A and $\overline{B}$ and between $\overline{A}$ and B, respectively. Finally, Fig. 9(g) and (h) highlight the contradictions between A and $\overline{A}$ and between B and $\overline{B}$, respectively. Taken together, these six relations constitute the so-called 'degenerate square of opposition' shown in Fig. 9(b). Apart from its two diagonals of contradiction, this Aristotelian diagram does not have any Aristotelian relations to visualize (because the four other pairs of sets are unconnected, i.e., do not stand in any Aristotelian relation).

4 Discussion and Future Research

In the previous section we have considered the seven relations between two (non-trivial) sets, and shown how the Euler diagrams for each of these seven relations systematically give rise to a well-defined Aristotelian diagram; cf. parts (a) and (b) of Figs. 3–9. The resulting Aristotelian diagrams turn out be of various types: we obtained four classical squares of opposition, but also two pairs of contradictories (PCDs) and one degenerate square of opposition. Using recent terminology from logical geometry, we say that these constitute three distinct Aristotelian families, which are pairwise not Aristotelian isomorphic [3, 9].

It turns out that these findings can be linked to other interesting notions from logical geometry, such as the information ordering on the seven relations, which was already mentioned in Sect. 2 (in particular, cf. Fig. 2), and also the notion of Boolean complexity of Aristotelian diagrams.[11] Specifically, we observe the following connections:

- the two *most informative* relations, i.e. contradiction and bi-implication, give rise to a *PCD* (cf. Figs. 7–8), which has a Boolean complexity of 2,
- the four *intermediately informative* relations, i.e. contrariety, subcontrariety, left-implication and right-implication, give rise to a *classical square of opposition* (cf. Figs. 3–6), which has a Boolean complexity of 3,
- the *least informative* relation, i.e. unconnectedness, gives rise to a *degenerate square of opposition* (cf. Fig. 9), which has a Boolean complexity of 4.

[11] A detailed discussion of the notion of Boolean complexity (or bitstring length) falls outside the scope of this paper. Very roughly, the idea is that the Boolean complexity of a diagram D is the smallest number n of bits that are required to faithfully encode D. Formally, given a Boolean algebra $\mathbb{B}$ and diagram $D = \{x_1, \ldots, x_n\}$, we have $n = |\{\pm x_1 \wedge_{\mathbb{B}} \cdots \wedge_{\mathbb{B}} \pm x_n \mid \pm x_1 \wedge_{\mathbb{B}} \cdots \wedge_{\mathbb{B}} \pm x_n \neq \bot_{\mathbb{B}}\}|$ (where $+x = x$ and $-x = \neg_{\mathbb{B}} x$); see [9] for much more mathematical details, motivation and examples.

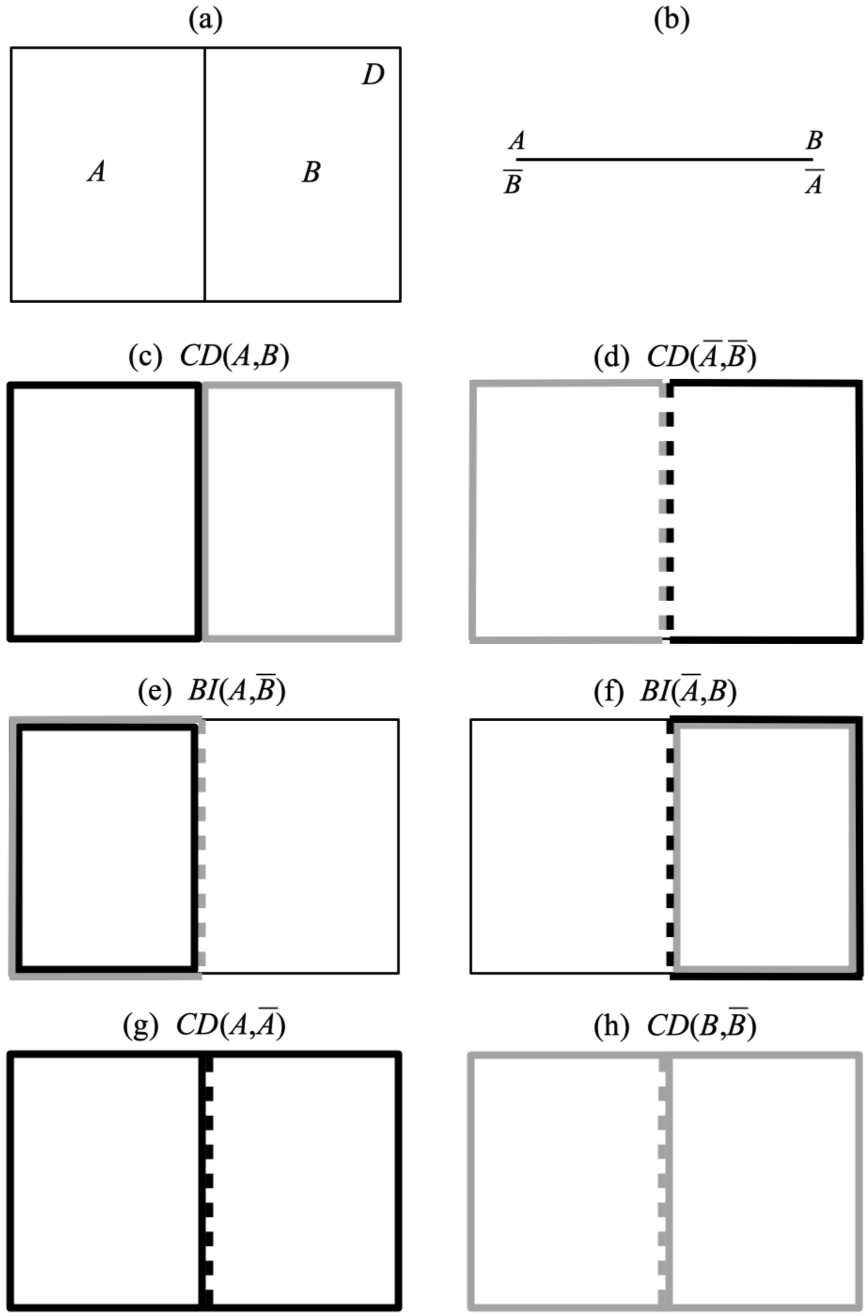

Fig. 8. (a) Euler diagram for the contradiction between A and B. (b) The corresponding PCD. (c–h) Highlighting the six relations among A, B, $\overline{A}$ and $\overline{B}$ in the Euler diagram (note: the four CD are Aristotelian relations, but the two BI are not).

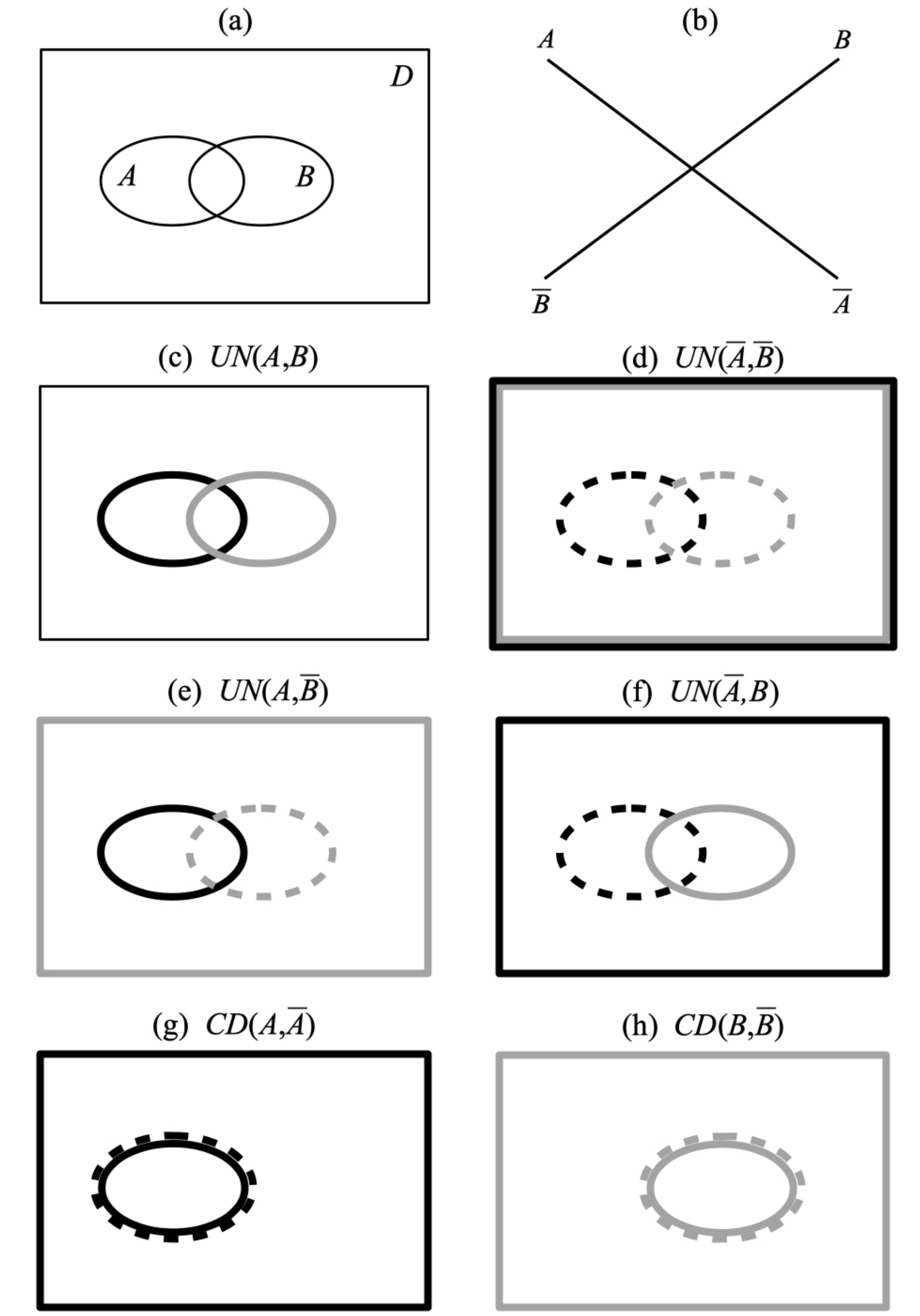

Fig. 9. (a) Euler diagram for the unconnectedness between A and B. (b) The corresponding degenerate square of opposition. (c–h) Highlighting the six relations among A, B, $\overline{A}$ and $\overline{B}$ in the Euler diagram (note: the two CD are Aristotelian relations, but the four UN are not).

We thus find an inverse correlation between (i) the information level of the relation visualized by the Euler diagram and (ii) the Boolean complexity of the corresponding Aristotelian diagram.

The true significance of these results is not yet fully understood at this point, but they clearly illustrate the theoretical fruitfulness of this approach within logical geometry. Furthermore, and even more importantly, by systematically linking Aristotelian diagrams with Euler diagrams, we have taken an important next step in charting the place of Aristotelian diagrams (and thus of logical geometry) within the broader landscape of logical diagrams research.[12] Since there exists a vast amount of work on diagrammatic reasoning with Euler diagrams, establishing a bridge to Aristotelian diagrams will hopefully inspire new, analogous work on diagrammatic reasoning with Aristotelian diagrams as well.

Thus far we have focused exclusively on (Aristotelian diagrams corresponding to) Euler diagrams for two non-trivial sets A and B. This suggests several avenues for further research; we finish this paper by mentioning three of them:

- What happens if we remove the restriction that the sets should be *non-trivial*, in other words, if we allow that $A = D$ or $A = \emptyset$ or $B = D$ or $B = \emptyset$? The Euler diagrams for these cases should be fairly straightforward, but the corresponding Aristotelian diagrams will violate the condition (which is usually considered to be fundamental in logical geometry) that Aristotelian diagrams should only contain non-trivial elements.
- What about Aristotelian diagrams corresponding to Euler diagrams for *more than two* sets? A special case is when these multiple sets constitute a partition of the domain of discourse; it is known that in this special case, the corresponding Aristotelian diagram will be a (strong) α-structure [5, 19]; however, there are currently no results yet about the general case.
- What about other types of diagrams, e.g. *spider diagrams* [15]? Can these also be transformed into well-defined Aristotelian diagrams?

References

1. Bernhard, P.: Euler-Diagramme: Zur Morphologie einer Repräsentationsform in der Logik. Mentis, Paderborn (2001)
2. Bernhard, P.: Visualizations of the square of opposition. Logica Universalis **2**, 31–41 (2008)
3. Demey, L.: Computing the maximal Boolean complexity of families of Aristotelian diagrams. Journal of Logic and Computation **28**, 1323–1339 (2018)
4. Demey, L.: Metalogic, metalanguage and logical geometry. Logique et Analyse **248**, 453–478 (2019)
5. Demey, L.: From Euler diagrams in Schopenhauer to Aristotelian diagrams in Logical Geometry. In: Lemanski, J. (ed.) Language, Logic, and Mathematics in Schopenhauer, pp. 181–205. Springer, Cham (2020)

[12] An earlier step in this direction was our paper [6], which explores the interaction between Aristotelian diagrams and Hasse diagrams.

6. Demey, L., Smessaert, H.: The relationship between Aristotelian and Hasse diagrams. In: Dwyer, T., Purchase, H., Delaney, A. (eds.) Diagrammatic Representation and Inference, pp. 213–227. LNCS 8578, Springer (2014)
7. Demey, L., Smessaert, H.: The interaction between logic and geometry in Aristotelian diagrams. In: Jamnik, M., Uesaka, Y., Elzer Schwartz, S. (eds.) Diagrammatic Representation and Inference. pp. 67–82. LNCS 9781, Springer (2016)
8. Demey, L., Smessaert, H.: Logical and geometrical distance in polyhedral Aristotelian diagrams in knowledge representation. Symmetry **9(10)**(204) (2017)
9. Demey, L., Smessaert, H.: Combinatorial bitstring semantics for arbitrary logical fragments. Journal of Philosophical Logic **47**, 325–363 (2018)
10. Demey, L., Smessaert, H.: Geometric and cognitive differences between Aristotelian diagrams for the Boolean algebra $\mathbb{B}_4$. Annals of Mathematics and Artificial Intelligence **83**, 185–208 (2018)
11. Edwards, A.W.F.: An eleventh-century Venn diagram. BSHM Bulletin **21**, 119–121 (2006)
12. Faris, J.: The Gergonne relations. Journal of Symbolic Logic **20**, 207–231 (1955)
13. Geudens, C., Demey, L.: On the Aristotelian roots of the modal square of opposition. Logique et Analyse **255**, 313–348 (2021)
14. Grattan-Guinness, I.: The Gergonne relations and the intuitive use of Euler and Venn diagrams. Int. J. of Math. Ed. in Science and Tech. **8**, 23–30 (1977)
15. Howse, J., Molina, F., Taylor, J., Kent, S., Gil, J.Y.: Spider diagrams: A diagrammatic reasoning system. J. of Visual Languages & Computing **12**, 299–324 (2001)
16. Lemanski, J.: Periods in the use of Euler-type diagrams. Acta Baltica Historiae et Philosophiae Scientiarum **5**, 50–69 (2017)
17. Lemanski, J.: Logic diagrams in the Weigel and Weise circles. History and Philosophy of Logic **39**, 3–28 (2018)
18. Lemanski, J.: Euler-type diagrams and the quantification of the predicate. Journal of Philosophical Logic **49**, 401–416 (2020)
19. Lemanski, J., Demey, L.: Schopenhauer's partition diagrams and logical geometry. In: Basu, A., et al. (eds.) Diagrammatic Representation and Inference. pp. 149–165. LNCS 12909, Springer (2021)
20. Linker, S.: Sequent calculus for Euler diagrams. In: Chapman, P., et al. (eds.) Diagrams 2018, pp. 399–407. LNCS 10871, Springer (2018)
21. Londey, D., Johanson, C.: Apuleius and the square of opposition. Phronesis **29**, 165–173 (1984)
22. Moktefi, A., Shin, S.J.: A history of logic diagrams. In: Gabbay, D.M., Pelletier, F.J., Woods, J. (eds.) Handbook of the History of Logic. Volume 11. Logic: A History of its Central Concepts, pp. 611–682. North-Holland, Amsterdam (2012)
23. Read, S.: John Buridan's theory of consequence and his octagons of opposition. In: Béziau, J.Y., Jacquette, D. (eds.) Around and Beyond the Square of Opposition, pp. 93–110. Springer, Basel (2012)
24. Rodgers, P.: A survey of Euler diagrams. Journal of Visual Languages & Computing **25**(3), 134–155 (2014)
25. Smessaert, H., Demey, L.: Logical geometries and information in the square of opposition. Journal of Logic, Language and Information **23**, 527–565 (2014)
26. Stapleton, G., Moktefi, A., Howse, J., Burton, J.: Euler diagrams through the looking glass: From extent to intent. In: Chapman, P., et al. (eds.) Diagrammatic Representation and Inference, pp. 365–381. LNCS 10871, Springer (2018)
27. Stapleton, G., Shimojima, A., Jamnik, M.: The observational advantages of Euler diagrams with existential import. In: Chapman, P., et al. (eds.) Diagrammatic Representation and Inference, pp. 313–329. LNCS 10871, Springer (2018)