Aristotelian and Duality Relations with Proportional Quantifiers

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The *Aristotelian* relations of contradiction (CD), contrariety (C), subcontrariety (SC) and subalternation (SA) have been argued to be conceptually independent of the *duality* relations of internal negation (IN), external negation (EN) and dual negation (DN) [1, 2, 4]. For any fragment of 4 formulas (from a logical language $\mathcal{L}$ for a logical system S) which is closed under negation — i.e. which consists of two pairs of contradictories — the former set of relations can be diagrammatically represented as a (possibly degenerate) *Aristotelian square*, whereas the latter set gives rise to a (possibly degenerate) *duality square*. Some such fragments only constitute an Aristotelian square — as is the case for the numerical quantifiers in Figure 1 —, whereas others yield both an Aristotelian and a duality square simultaneously — as is the case for the quantifiers of Standard Predicate Logic in Figure 2. The

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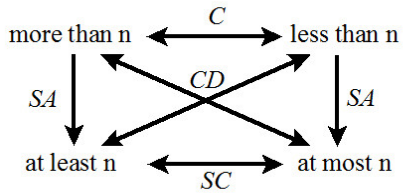
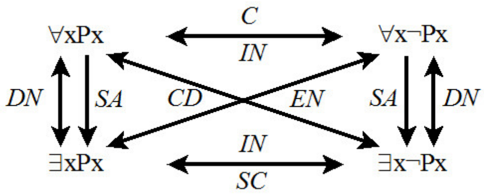


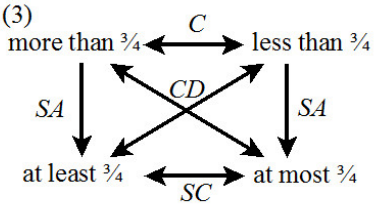
(2)



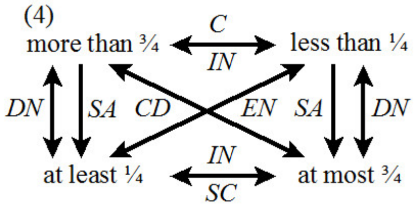
set of Aristotelian relations is fundamentally *hybrid*: (i) CD, C and SC are symmetric and defined in terms of being true/false together, whereas SA is not symmetric and defined in terms of truth propagation [5]; and (ii) CD is a functional relation, but C, SC and SA are not. All duality relations, by contrast, are symmetric and functional. A further mismatch concerns the fact that the single duality relation of IN seems to correspond to two Aristotelian relations, viz. either C or SC. On a more abstract level, Aristotelian relations have been shown to be highly logic-sensitive, whereas duality relations are insensitive to the underlying logic [2, 5].

The central aim of the presentation is to chart which of the above logical relations hold between quantificational formulas expressing the notion of *proportionality*. Two types of expressions will be distinguished: (i) *explicit proportionals* such as *at least two thirds of the A's are B* or *less than 20 percent of the A's are B*, in which the proportion is explicitly referred to in terms of fractions or percentages; and (ii) *implicit proportionals* such as *a minority/majority of the A's are B*, in which the actual proportion remains implicit. Explicit proportionals will be

(3)



(4)



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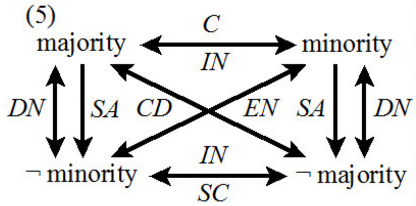

argued to give rise to (at least) two constellations: (i) the square in Figure 3 corresponds to that in Figure 1 in being an Aristotelian square only, wheras (ii) the square in Figure 4 corresponds to that in Figure 2 in being both an Aristotelian and a duality square. Implicit proportionals, then, automatically yield 'double' squares, as in Figure 5. The analysis is carried out withing the framework of Logical Geometry (`www.logicalgeometry.org`) and makes use of so-called *bitstring* representations, which are compact combinatorial representations of the denotations of the various types of proportional expressions that are based on (scalar) partitionings of logical space. Finally, since these proportional expressions are generalised quantifiers, their monotonicity properties will also be studied [3].

References

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