One-sided versus two-sided readings of *many* and *few*

Hans Smessaert and Lorenz Demey

LNAT3, February 5-6, 2015, Brussels
Aims of this talk:

- discuss Béziau’s (unpublished LNAT1) proposal to transpose his results on the logical geometry of the modal logic S5 to that of the subjective quantifiers *many* and *few*

- propose an alternative analysis of *many* and *few*, which seems to fare equally well from a strictly logical perspective, but which we argue to be more in line with certain linguistic desiderata

- compare the two analyses in terms of two scales:
  - scale of semantic complexity
  - scale of lexical complexity

- compare the two analyses in terms of the types of Aristotelian diagrams they generate
Introduction

One-sided readings of “many” and “few”

Two-sided readings of “many” and “few”

Semantic versus lexical complexity

Aristotelian diagrams for “many” and “few”

Conclusion
Overview

1 Introduction

2 One-sided readings of “many” and “few”

3 Two-sided readings of “many” and “few”

4 Semantic versus lexical complexity

5 Aristotelian diagrams for “many” and “few”

6 Conclusion
The analogy between S5-formulas and FOL-quantifiers

<table>
<thead>
<tr>
<th>S5-formula</th>
<th>bitstring</th>
<th>FOL-quantifier</th>
</tr>
</thead>
<tbody>
<tr>
<td>□p</td>
<td>100</td>
<td>all</td>
</tr>
<tr>
<td>¬□p</td>
<td>011</td>
<td>not all</td>
</tr>
<tr>
<td>¬◊p</td>
<td>001</td>
<td>no</td>
</tr>
<tr>
<td>◊p</td>
<td>110</td>
<td>at least one</td>
</tr>
<tr>
<td>□p ∨ ¬◊p</td>
<td>101</td>
<td>no or all</td>
</tr>
<tr>
<td>¬□p ∧ ◊p</td>
<td>010</td>
<td>some</td>
</tr>
</tbody>
</table>

\[
\text{some} \equiv \text{at least one but not all} \quad 010 = 110 \land 011 \\
\text{at least one} \equiv \text{some or all} \quad 110 = 010 \lor 100
\]

One-sided versus two-sided *many* and *few* – H. Smessaert & L. Demey
Béziau’s one-sided readings of “many” and “few”

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<th>FOL-quantifier</th>
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<tr>
<td>□p</td>
<td>1000</td>
<td>all</td>
</tr>
<tr>
<td>¬□p</td>
<td>0111</td>
<td>not all</td>
</tr>
<tr>
<td>¬◊p</td>
<td>0001</td>
<td>no</td>
</tr>
<tr>
<td>◊p</td>
<td>1110</td>
<td>at least one</td>
</tr>
<tr>
<td>□p ∨ ¬◊p</td>
<td>1001</td>
<td>no or all</td>
</tr>
<tr>
<td>¬□p ∧ ◊p</td>
<td>0110</td>
<td>some</td>
</tr>
<tr>
<td>p</td>
<td>1100</td>
<td>many₁</td>
</tr>
<tr>
<td>¬p</td>
<td>0011</td>
<td>few₁</td>
</tr>
</tbody>
</table>
Béziau’s one-sided readings of “many” and “few”

<table>
<thead>
<tr>
<th>level</th>
<th>S5-formula</th>
<th>bitstring</th>
<th>subjective quantifier</th>
</tr>
</thead>
<tbody>
<tr>
<td>L2</td>
<td>$p$</td>
<td>1100</td>
<td>$many_1$</td>
</tr>
<tr>
<td></td>
<td>$\neg p$</td>
<td>0011</td>
<td>$few_1$</td>
</tr>
<tr>
<td>L1</td>
<td>$p \land \neg \Box p$</td>
<td>0100</td>
<td>$many_1 \text{ but not all}$</td>
</tr>
<tr>
<td></td>
<td>$\neg p \land \Diamond p$</td>
<td>0010</td>
<td>$\text{at least one but few}_1$</td>
</tr>
<tr>
<td>L3</td>
<td>$\neg p \lor \Box p$</td>
<td>1011</td>
<td>$all \lor few_1$</td>
</tr>
<tr>
<td></td>
<td>$p \lor \neg \Diamond p$</td>
<td>1101</td>
<td>$no \lor many_1$</td>
</tr>
<tr>
<td>L2</td>
<td>$\Box p \lor (\neg p \land \Diamond p)$</td>
<td>1010</td>
<td>$all \lor (\text{at least one but few}_1)$</td>
</tr>
<tr>
<td></td>
<td>$\neg \Box p \land (p \lor \neg \Diamond p)$</td>
<td>0101</td>
<td>$no \lor (many_1 \text{ but not all})$</td>
</tr>
</tbody>
</table>

The conjunctions $many_1 \text{ but not all}$ and $\text{at least one but few}_1$ create the L1 elements 0100 and 0010 by excluding the extreme values of the tripartition, i.e. all (1000) and no (0001), respectively.
Problems with Béziau’s one-sided readings

- entailments in S5
  - from L1 ‘necessity’ (1000) to L2 ‘actual truth’ (1100)
  - from L1 ‘impossibility’ (0001) to L2 ‘actual falsehood’ (0011)
- analogous entailments for subjective quantifiers
  - from L1 all (1000) to L2 many₁ (1100)
  - from L1 no (0001) to L2 few₁ (0011)
- suppose that John has read all three books in the universe of discourse
  - John has read all books is obviously true
  - John has read many books is very likely to be considered false
    (‘three books’ does not really count as ‘many books’)
- suppose that John has read none of the books in the universe of discourse
  - John has read no books is obviously true
  - John has read few books is much less obvious
    (conflict with the existential presupposition of few)

- solution: two-sided readings for few and many
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4. Semantic versus lexical complexity
5. Aristotelian diagrams for “many” and “few”
6. Conclusion
Two-sided readings of “many” and “few”

Many

\[ \text{many}_2 = 0100 = \text{two-sided L1 incompatible with } \text{all} = 1000 \]

\[ \text{few}_2 = 0010 = \text{two-sided L1 incompatible with } \text{no} = 0001 \]

level 2 disjunctions = lexically complex expressions, cfr. English little or no; Dutch weinig of geen and French peu ou pas

\[ \text{many}_2 \text{ or all}/\text{many}_2 \text{ if not all} \]
\[ 0100 \lor 1000 = 1100 \equiv \text{many}_1 \]

\[ \text{few}_2 \text{ or no}/\text{few}_2 \text{ if any} \]
\[ 0010 \lor 0001 = 0011 \equiv \text{few}_1 \]

\[ \text{many}_2 \text{ or few}_2 \]
\[ 0100 \lor 0010 = 0110 \equiv \text{some} \]
Two-sided readings of “many” and “few”

<table>
<thead>
<tr>
<th>level</th>
<th>Béziau’s analysis</th>
<th>bitstring</th>
<th>alternative analysis</th>
</tr>
</thead>
<tbody>
<tr>
<td>L2</td>
<td>( \text{many}_1 )</td>
<td>1100</td>
<td>( \text{many}_2 ) if not all</td>
</tr>
<tr>
<td></td>
<td>( \text{few}_1 )</td>
<td>0011</td>
<td>( \text{few}_2 ) if any</td>
</tr>
<tr>
<td>L1</td>
<td>( \text{many}_1 ) but not all</td>
<td>0100</td>
<td>( \text{many}_2 )</td>
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<td>( \text{few}_2 )</td>
</tr>
<tr>
<td>L3</td>
<td>( \text{all or few}_1 )</td>
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<td>( \text{all or (few}_2 ) if any)</td>
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<tr>
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<td>( \text{no or (many}_1 ) but not all)</td>
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<td>( \text{no or many}_2 )</td>
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**discrepancies** between:
- semantic complexity (full line arrows) = entailment L1 > L2 > L3
- lexical complexity (dashed line arrows) = amount of lexical material

**difference in orientation** between:
- the lattices for semantic complexity = from top to bottom
- the lattices for lexical complexity = from the outside inwards
Semantic vs lexical complexity in the alternative analysis

- no more discrepancies between:
  - semantic complexity (full line arrows) = entailment \( L1 > L2 > L3 \)
  - lexical complexity (dashed line arrows) = amount of lexical material

- parallel orientation of:
  - the lattices for semantic complexity = from top to bottom
  - the lattices for lexical complexity = from top to bottom
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### Strong Jacoby-Sesmat-Blanché hexagons

- **contradiction**: 3 diagonals: \(2 \times \text{L1-L3} \) and \(1 \times \text{L2-L2}\)
- **contrariety**: triangle \(\text{L1-L2-L1}\)
- **subcontrariety**: triangle \(\text{L3-L2-L3}\)
- **subalternation**: 6 arrows: \(2 \times \text{L1-L2}, 2 \times \text{L2-L3}\) and \(2 \times \text{L1-L3}\)
**contradiction**: 2 x L1-L3 and 2 x L2-L2 \(\leadsto\) `many_1/few_1`

**contrariety**: 1 x L1-L1 and 4 x L1-L2 \(\leadsto\) `many_2/few_2`

**subcontrariety**: 1 x L3-L3 and 4 x L2-L3

**subalternation**: 4 transitivity triangles L1-L2-L3

**unconnectedness square**: 4 pairs of L2-L2
The Rhombic dodecahedron RDH = cube + octahedron

- Cube
  - 8 vertices
  - 4 x L1
  - 4 x L3

- Octahedron
  - 6 vertices
  - 6 x L2

- Rhombic dodecahedron
  - 14 vertices
  - *L0 *0000
  - *L4 *1111
  - \(14 = 2^4 - 2 = 16 - 2\)

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Complementarity of JSB hexagon and Buridan rhombicube

strong JSB hexagon

Buridan octagon

rhombic dodecahedron

rhombicube

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discussed Béziau’s one-sided analysis of many₁/few₁ based on the analogy between the modal logic S5 and the subjective quantifiers.

proposed an alternative, two-sided analysis of many₂/few₂, which more adequately reflects the relations of entailment (all → many, and no → few) and disjunction (few if any, many if not all).

compared the two analyses in terms of discrepancies between the scale of semantic complexity and the scale of lexical complexity.

compared the two analyses in terms of the types of Aristotelian diagrams they generate: identical strong Jacoby-Sesmat-Blanché hexagons but different Buridan octagons/rhombicubes:

- **contradiction** for many₁/few₁
- **contrariety** for many₂/few₂
Thank you!

More info: www.logicalgeometry.org