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Tutorial: An Introduction to Logical Geometry: part I

Decorations and bitstrings

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Fifth World Congress on the Square of Oppositions

Easter Island, November 2016

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Central aim of Logical Geometry

The central aim of Logical Geometry (**www.logicalgeometry.org**) is

- to develop an *interdisciplinary framework*
- for the study of *logical diagrams*
- in the analysis of *logical, linguistic and conceptual systems*.

More in particular we:

- analyse logical relations of opposition, implication and duality between expressions in various **logical, linguistic** & **conceptual** systems.
- study the **logical diagrams** from the perspective of:
  - their *abstract-logical* properties
  - their *visual-geometric* properties
- develop an **interdisciplinary framework** integrating insights from logic, formal semantics, algebra, group theory, lattice theory, computer graphics, cognitive psychology, information visualisation and diagrams design.

Overview

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- Hans Smessaert (1993). *The Logical Geometry of Comparison and Quantification. A cross-categorial analysis of Dutch determiners and aspectual adverbs*. PhD in linguistics, KU Leuven, Belgium.
- Lorenz Demey (2014). *Believing in Logic and Philosophy*. PhD in logic and analytic philosophy, KU Leuven, Belgium.
- Koen Roelandt (2016). *Most or the Art of Compositionality: Dutch de/het meeste at the Syntax-Semantics Interface*. PhD in linguistics, KU Leuven, Belgium.

- Closely related PhD dissertations:
  - Dany Jaspers (2005). *Operators in the Lexicon. On the Negative Logic of Natural Language*. PhD in linguistics, Leiden University, The Netherlands.
  - Alessio Moretti (2009). *The geometry of logical opposition*. PhD in logic, University of Neuchâtel, Switzerland.
- World Congress on the Square of Opposition:
  - Square 2007: Montreux, Switzerland
  - Square 2010: Corte, Corsica
  - Square 2012: Beirut, Lebanon
  - Square 2014: Vatican, Roma
  - Square 2016: Easter Island, Chile
- International Conference on the Theory and Application of Diagrams:
  - Diagrams 2012: Canterbury, UK
  - Diagrams 2014: Melbourne, Australia
  - Diagrams 2016: Philadelphia, US

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- Part I: Decorations and bitstrings
  - Bitstrings: the basics
  - Decorations: applications in logic, linguistics and cognition
  - Bitstrings anno 2016
  - Summary Part I: Interdisciplinarity of LG
- Part II: Abstract-logical properties of diagrams
  - Logic: opposition relations versus implication relations
  - Logic: logic-sensitivity and Boolean subtypes
  - Logic: Aristotelian relations versus duality relations
  - Summary Part II: Interdisciplinarity of LG
- Part III: Visual-geometric properties of diagrams
  - Geometry: projections
  - Geometry: subdiagrams and complementarity
  - Geometry: diagram design principles
  - Summary Part III: Interdisciplinarity of LG

Structure of the tutorial: part I

1. Introduction
   - Central aim of Logical Geometry
   - Background of Logical Geometry
   - Structure of the tutorial
2. Bitstrings in LG: the basics
3. Decorations: applications in logic, linguistics and cognition
   - Decorations: modal logic S5
   - Decorations: subjective quantifiers
   - Decorations: gradable adjectives and colour terms
4. Bitstrings anno 2016
   - Earlier results and limitations
   - The bitstring technique
   - New decorations: public announcements, definite descriptions
5. Summary Part I

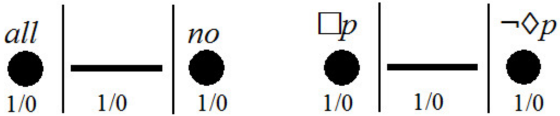
Bitstrings in LG: the basics

- Bitstrings are sequences of bits (0/1) that encode the denotations of formulas or expressions from:
  - **logical systems**: e.g. classical propositional logic, first-order logic, modal logic and public announcement logic
  - **lexical fields**: e.g. comparative quantification, subjective quantification, color terms and set inclusion relations

- Remark:
  - we use bitstrings to encode **formulas**, not **relations** between formulas
  - if a formula φ is encoded by the bitstring b, we write $\beta(\varphi) = b$

- Each bit provides an answer to a (binary) meaningful question (analysis of generalized quantifiers as sets of sets).

- Each question concerns a component (point or interval) of a scalar structure creating a partition of logical space:



- In Predicate Logic/GQT: Is R(A,B) true if

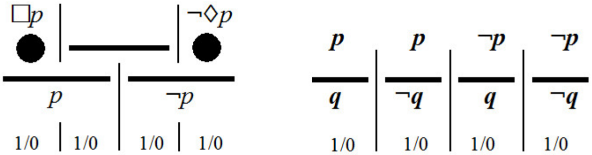
$A \subseteq B$	yes/no
$A \not\subseteq B$ and $A \cap B \neq \emptyset$	yes/no
$A \cap B = \emptyset$	yes/no

- Examples:

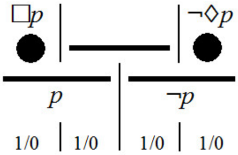
β(*All A are B*)	$= 100$	$= \langle$ yes, no, no $\rangle$
β(*Some but not all A are B*)	$= 010$	$= \langle$ no, yes, no $\rangle$
β(*Not all A are B*)	$= 011$	$= \langle$ no, yes, yes $\rangle$

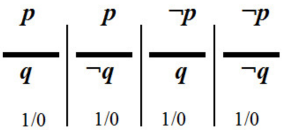
- In Modal Logic: Is φ true if
  - p is true in all possible worlds? yes/no
  - p is true in some but not in all possible worlds? yes/no
  - p is true in no possible worlds? yes/no
- Examples:
  - $\beta(\Diamond p) = 110 = \langle$ yes, yes, no $\rangle$
  - $\beta(\Diamond p \wedge \Diamond \neg p) = 010 = \langle$ no, yes, no $\rangle$
  - $\beta(\Diamond \neg p) = 011 = \langle$ no, yes, yes $\rangle$

Modal Logic	GQT	level 1/0	level 2/3	GQT	Modal Logic
necessary ($\Box p$)	*all*	100	011	*not all*	*not necessary* ($\neg\Box p$)
contingent ($\neg\Box p \wedge \Diamond p$)	*some but not all*	010	101	*no or all*	*not contingent* ($\Box p \vee \neg\Diamond p$)
impossible ($\neg\Diamond p$)	*no*	001	110	*some*	*possible* ($\Diamond p$)
contradiction ($\Box p \wedge \neg\Box p$)	*some and no*	000	111	*some or no*	*tautology* ($\Box p \vee \neg\Box p$)



- In Modal Logic S5: Is φ true if:
  - p is true in all possible worlds W? yes/no
  - p is true in the actual world but not in all possible Ws? yes/no
  - p is true in some possible Ws but not in the actual world? yes/no
  - p is true in no possible worlds W? yes/no
- Examples:
  - $\beta(\Diamond p) = 1110 = \langle$ yes, yes, yes, no $\rangle$
  - $\beta(\Diamond p \wedge \Diamond \neg p) = 0110 = \langle$ no, yes, yes, no $\rangle$
  - $\beta(\Diamond \neg p) = 0111 = \langle$ no, yes, yes, yes $\rangle$





- In Propositional Logic: Is φ true if:
  - p is true and q is true? yes/no
  - p is true and q is false? yes/no
  - p is false and q is true? yes/no
  - p is false and q is false? yes/no
- Examples:

$$\beta(\neg p) = 0011 = \langle \text{ no, no, yes, yes } \rangle$$

$$\beta(p \leftrightarrow q) = 1001 = \langle \text{ yes, no, no, yes } \rangle$$

$$\beta(p \rightarrow q) = 1011 = \langle \text{ yes, no, yes, yes } \rangle$$

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$2^3 = 8$ bitstrings of length 3 $\rightsquigarrow$ $2^4 = 16$ bitstrings of length 4

Modal Logic S5	Propositional Logic	bitstrings level 1	bitstrings level 3	Propositional Logic	Modal Logic S5
$\Box p$	$p \wedge q$	1000	0111	$\neg(p \wedge q)$	$\neg\Box p$
$\neg\Box p \wedge p$	$\neg(p \rightarrow q)$	0100	1011	$p \rightarrow q$	$\Box p \vee \neg p$
$\Diamond p \wedge \neg p$	$\neg(p \leftarrow q)$	0010	1101	$p \leftarrow q$	$\neg\Diamond p \vee p$
$\neg\Diamond p$	$\neg(p \vee q)$	0001	1110	$p \vee q$	$\Diamond p$

Modal Logic S5	Propositional Logic	bitstrings level 2/0	bitstrings level 2/4	Propositional Logic	Modal Logic S5
p	p	1100	0011	$\neg p$	$\neg p$
$\Box p \vee (\Diamond p \wedge \neg p)$	q	1010	0101	$\neg q$	$\neg\Diamond p \vee (\neg\Box p \wedge p)$
$\Box p \vee \neg\Diamond p$	$p \leftrightarrow q$	1001	0110	$\neg(p \leftrightarrow q)$	$\neg\Box p \wedge \Diamond p$
$\Box p \wedge \neg\Box p$	$p \wedge \neg p$	0000	1111	$p \vee \neg p$	$\Box p \vee \neg\Box p$

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- Relative to a logical system S, two **formulas** φ, ψ are

contradictory (CD)	iff	$S \models \neg(\varphi \wedge \psi)$	and	$S \models \neg(\neg\varphi \wedge \neg\psi)$
contrary (C)	iff	$S \models \neg(\varphi \wedge \psi)$	and	$S \not\models \neg(\neg\varphi \wedge \neg\psi)$
subcontrary (SC)	iff	$S \not\models \neg(\varphi \wedge \psi)$	and	$S \models \neg(\neg\varphi \wedge \neg\psi)$
in subalternation (SA)	iff	$S \models \varphi \to \psi$	and	$S \not\models \psi \to \varphi$

- In terms of bitstrings, two **bitstrings** b_1 and b_2 are

contradictory (CD)	iff	$b_1 \wedge b_2 = 0 \cdots 0$	and	$b_1 \vee b_2 = 1 \cdots 1$
contrary (C)	iff	$b_1 \wedge b_2 = 0 \cdots 0$	and	$b_1 \vee b_2 \neq 1 \cdots 1$
subcontrary (SC)	iff	$b_1 \wedge b_2 \neq 0 \cdots 0$	and	$b_1 \vee b_2 = 1 \cdots 1$
in subalternation (SA)	iff	$b_1 \wedge b_2 = b_1$	and	$b_1 \vee b_2 \neq b_1$

- φ and ψ stand in some Aristotelian relation (defined for S) iff $\beta(\varphi)$ and $\beta(\psi)$ stand in that same relation (defined for bitstrings).
- β maps formulas from S to bitstrings, preserving Aristotelian structure (Representation Theorem for finite Boolean algebras)

Overview

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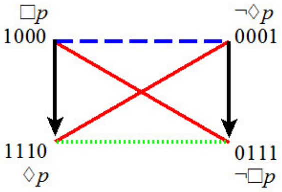
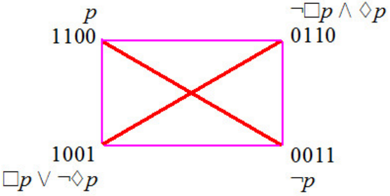


classical square



degenerate square

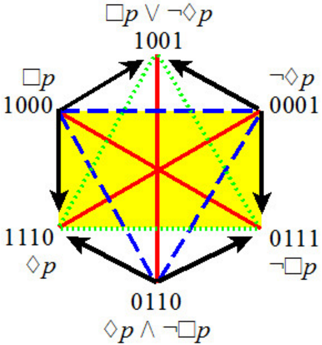
2 contradictories (CD)
2 PCDs (L1-L3)

2 contradictories (CD)
2 PCDs (L2-L2)

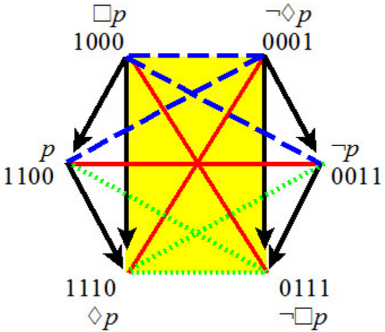
2 subalternations (SA)
1 contrariety (C)
1 subcontrariety (SC)

4 × unconnectedness (U)

Decorations: hexagons in modal logic S5

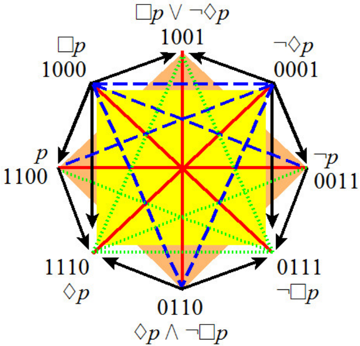


Jacoby-Sesmat-Blanché hexagon
3 PCDs
6 subalternations (SA)
3 contrarieties (C)
3 subcontrarieties (SC)

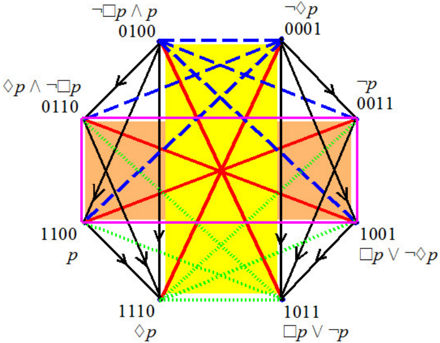


Sherwood-Czezowski hexagon
3 PCDs
6 subalternations (SA)
3 contrarieties (C)
3 subcontrarieties (SC)

Decorations: octagons in modal logic S5

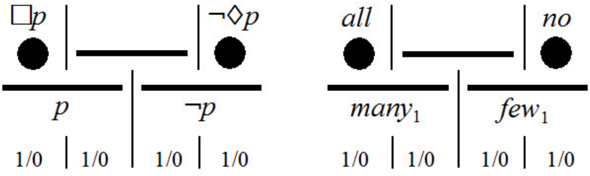


Béziau octagon
4 PCDs
10 SAs & 5 Cs & 5 SCs
4 × unconnectedness (U)



Buridan octagon
4 PCDs
10 SAs & 5 Cs & 5 SCs
4 × unconnectedness (U)

Overview



level	S5-formula	bitstring	subjective quantifier
L2	p	1100	*many*$_1$
	$\neg p$	0011	*few*$_1$
L1	$p \wedge \neg\Box p$	0100	*many*$_1$ *but not all*
	$\neg p \wedge \Diamond p$	0010	*at least one but few*$_1$

The conjunctions *many*$_1$ *but not all* and *at least one but few*$_1$ create the L1 elements 0100 and 0010 by excluding the extreme values of the tripartition, i.e. *all* (1000) and *no* (0001), respectively.

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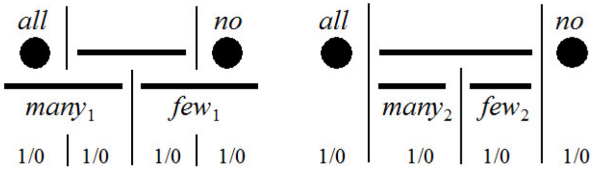
- entailments in S5
  - from L1 'necessity' (1000) to L2 'actual truth' (1100)
  - from L1 'impossibility' (0001) to L2 'actual falsehood' (0011)
- analogous entailments for subjective quantifiers
  - from L1 *all* (1000) to L2 *many*$_1$ (1100)
  - from L1 *no* (0001) to L2 *few*$_1$ (0011)
- suppose that John has read all three books in the universe of discourse
  - *John has read all books* is obviously true
  - *John has read many books* is very likely to be considered false ('three books' does not really count as 'many books')
- suppose that John has read none of the books in the univ. of discourse
  - *John has read no books* is obviously true
  - *John has read few books* is much less obvious (conflict with the existential presupposition of *few*)

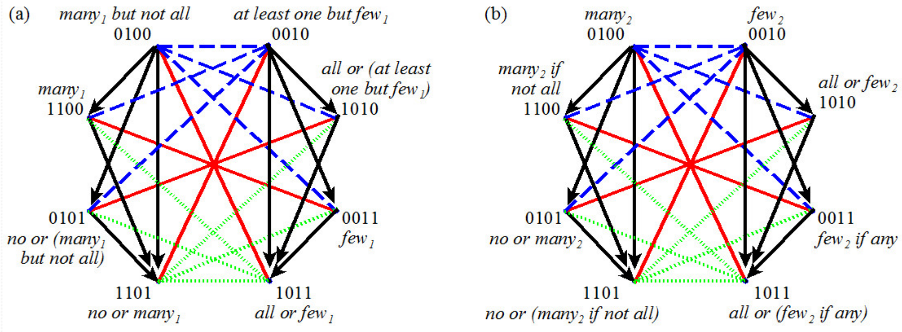
- **solution**: two-sided readings for *few* and *many*



level	Béziau's analysis	bitstring	alternative analysis
L2	*many*$_1$	1100	*many*$_2$ *if not all*
	few$_1$	0011	*few*$_2$ *if any*
L1	*many*$_1$ *but not all*	0100	*many*$_2$
	at least one but few$_1$	0010	*few*$_2$

level 2 **disjunctions** = lexically complex expressions,
cfr. English *little or no*; Dutch *weinig of geen* and French *peu ou pas*

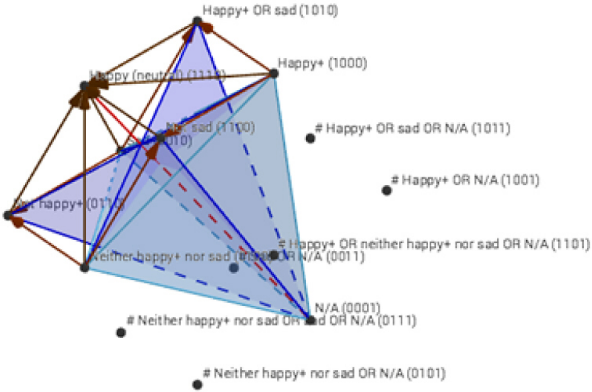
Decorations: alternative two-sided analysis of subjective Qs



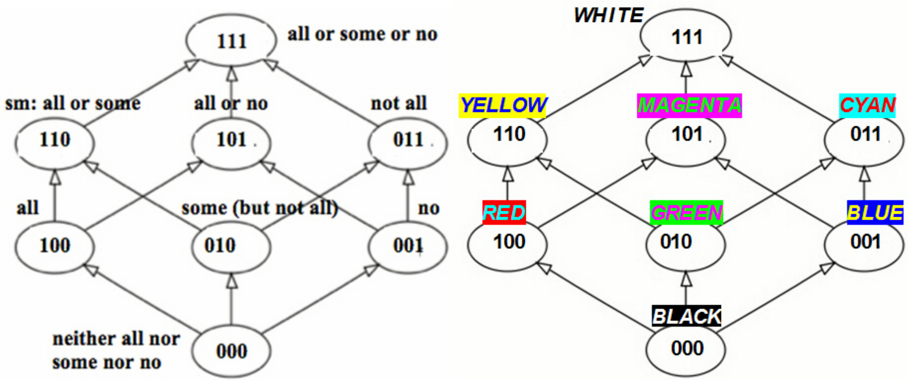
- **contradiction**: 2 x L1-L3 and 2 x L2-L2 ⇝ ***many$_1$/few$_1$***
- **contrariety**: 1 x L1-L1 and 4 x L1-L2 ⇝ ***many$_2$/few$_2$***
- **subcontrariety**: 1 x L3-L3 and 4 x L2-L3
- **subalternation**: 4 transitivity triangles L1-L2-L3
- **unconnectedness square**: 4 pairs of L2-L2

Decorations: gradable adjectives

Happy$^{\pm}$				
Happy^{+}	Neither happy^{+} nor sad	Sad	N/A	
1	0	0	0	Happy^{+}
0	1	0	0	Neither happy^{+} nor sad
0	0	1	0	Sad
0	0	0	1	N/A
1	1	0	0	Happy^{+} OR neither happy^{+} nor sad
1	0	1	0	Happy^{+} OR sad
0	1	1	0	Neither happy^{+} nor sad OR sad
1	1	1	0	Happy$^{\pm}$
1	0	0	1	Happy^{+} OR N/A
0	1	0	1	Neither happy^{+} nor sad OR N/A
0	0	1	1	Sad OR N/A
1	1	0	1	Happy^{+} OR neither happy^{+} nor sad OR N/A
1	0	1	1	Happy^{+} OR sad OR N/A
0	1	1	1	Neither happy^{+} nor sad OR sad OR N/A



Koen Roelandt (2016). *Most or the Art of Compositionality: Dutch de/het meeste at the Syntax-Semantics Interface*. PhD in linguistics, KU Leuven.



Dany Jaspers (2012). Logic and colour. *Logica Universalis* 6, 227–48.

Unconnectedness (logical independence):

- absence of any Aristotelian relation
- φ and ψ are unconnected iff:
  - φ and ψ may be true together
  - φ and ψ may be false together
  - φ does not entail ψ
  - ψ does not entail φ

- Unconnectedness requires bitstrings of length at least 4
- Theorem: φ and ψ unconnected $\Rightarrow \beta(\varphi)$ and $\beta(\psi)$ have ≥ 4 bits

- More details in Tutorial Part II (on informativity and opposition relations versus implication relations)

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Calculating (sub)contraries

- For any bitstring of length n and level i we can use simple combinatorial arguments to calculate the number of:

contradictories	$\#CD$	$= 1$
contraries	$\#C$	$= 2^{n-i} - 1$
subcontraries	$\#SC$	$= 2^{i} - 1$
non-contradictories	$\#NCD$	$= (2^{n-i} - 1)(2^{i} - 1)$

- Note that $\#CD < \#C, \#SC < \#NCD$ iff $1 < i < n - 1$
- Note that if $i \approx \frac{n}{2}$, then $\#C \approx \#SC$
- Bitstrings in middle levels have similar numbers of contraries and subcontraries
- For the relevance of these observations see Tutorial Part II (on informativity and opposition relations versus implication relations)

Use bitstrings to study embeddings

- Boolean closure of bitstrings length 4 $\stackrel{2009}{\Longrightarrow}$ rhombic dodecahedron (RDH)
- rhombic dodecahedron $\sim$ bitstrings of length 4
  strong JSB hexagon $\sim$ bitstrings of length 3
- compression of bitstrings: length 4 $\rightsquigarrow$ length 3
- e.g. $b_1 = b_2$: **11**00 $\rightsquigarrow$ **1**00, **00**10 $\rightsquigarrow$ **0**10, **00**11 $\rightsquigarrow$ **0**11
- 6 strong JSB hexagons in RDH $\sim$ 6 compressions length 4 $\rightsquigarrow$ length 3
$b_2 = b_3$,	$b_1 = b_2$,	$b_3 = b_4$,	$b_1 = b_4$,	$b_1 = b_3$,	$b_2 = b_4$
(1950s)	(2003)	(2003)	(2005*)	(2005)	(2005)

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How many hexagons can be constructed with bitstrings of length ℓ?

- 2^ℓ bitstrings of length $\ell \rightsquigarrow (2^\ell - 2)$ contingent bitstrings of length ℓ
- bitstrings are chosen in contradictory pairs: $\frac{(2^\ell-2)(2^\ell-4)(2^\ell-6)}{48}$
-

$\ell = 3$	$\ell = 4$	$\ell = 5$	$\ell = 6$	$\ell = 7$
$\frac{(\mathbf{6})(4)(2)}{48}$	$\frac{(\mathbf{14})(12)(10)}{48}$	$\frac{(\mathbf{30})(28)(26)}{48}$	$\frac{(\mathbf{62})(60)(58)}{48}$	$\frac{(\mathbf{126})(124)(122)}{48}$
1	35	455	4495	39711

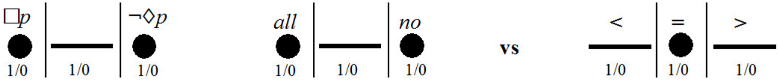
- computational importance of bitstrings for generating hexagons.

Different types of hexagons require bitstrings of different length:

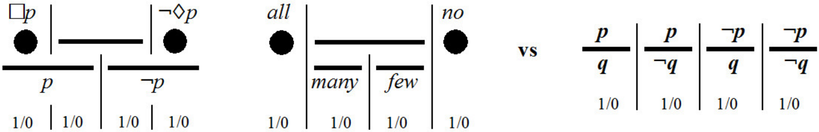
- strong Jacoby-Sesmat-Blanché (JSB) requires length 3
- weak JSB, Sherwood-Czezowski, U4 and U12 require length 4
- U8 requires length 5
- no hexagons require length 6, 7 ...

- Bitstrings generate new questions about
  - the linguistic/cognitive aspects of the expressions they encode
  - the relative weight/strength of individual bit positions inside bitstrings
  - the underlying scalar/linear structure of the conceptual domain

- Edges versus center in bitstrings of length 3



- Bitstrings of length 4 as refinements/expansions of bitstrings of length 3



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- From mathematical/algebraic perspective no difference (so far) between
  - 'linear' bitstrings (such as 1010)
  - 'non-linear' bitstrings (such as $1^0_1 0$)

- From linguistic/cognitive perspective difference is relevant :
  - Linear bitstrings imply that all questions (all bits) about a lexical field can be situated on a single dimension
    $\rightsquigarrow$ comparative quantification, proportional quantification, propositional connectives, *all*/*many*$_2$/*few*$_2$/*no*
  - Non-linear bitstrings imply that the various questions belong to fundamentally distinct dimensions
    $\rightsquigarrow$ modality in $S5$, *all*/*John*/*not-John*/*no*, *all*/*many*$_1$/*few*$_1$/*no*
  - Formulate empirical hypotheses concerning the cognitive complexity (e.g. processing times) of these lexical fields.
    $\rightsquigarrow$ future research

- It is not always clear how 'sensitive' bitstrings are to the specific properties of the underlying logical system: two formulas may enter into different Aristotelian relations with one another depending on the logical system and should therefore be assigned different bitstrings accordingly.
- The complex interplay between Boolean and Aristotelian structure requires further investigation: some fragments which have an isomorphic Aristotelian structure may nevertheless not be isomorphic from a Boolean point of view.
- The current approach does not provide a systematic strategy for establishing a bitstring semantics for any fragment $\mathcal{F}$ of any logical system S (e.g. formulas from Public Announcement Logic or the multi-operator formulas in Avicenna/Buridan)

$\Rightarrow$ **develop a more mathematically mature version of bitstring semantics that is able to overcome these different limitations**

Partitions Induced by Logical Fragments

- Let S be a logical system, and let $\mathcal{F} = \{\varphi_1, \ldots, \varphi_m\} \subseteq \mathcal{L}_\mathsf{S}$ be a finite fragment of the language of S.

- The *partition of* S *induced by* $\mathcal{F}$ is

$$\Pi_\mathsf{S}(\mathcal{F}) := \{\alpha \in \mathcal{L}_\mathsf{S} \mid \alpha \equiv_\mathsf{S} \pm\varphi_1 \wedge \cdots \wedge \pm\varphi_m, \text{ and } \alpha \text{ is S-consistent}\}.$$

  In this definition, $\pm\varphi$ stands for either φ or $\neg\varphi$. Furthermore, the formulas $\alpha \in \Pi_\mathsf{S}(\mathcal{F})$ will be called *anchor formulas*. They are:
  - mutually exclusive: $\mathsf{S} \models \neg(\alpha_i \wedge \alpha_j)$ for distinct $\alpha_i, \alpha_j \in \Pi_\mathsf{S}(\mathcal{F})$
  - jointly exhaustive: $\mathsf{S} \models \bigvee \Pi_\mathsf{S}(\mathcal{F})$

- Each anchor formula is thus equivalent to a conjunction consisting of $m = |\mathcal{F}|$ conjuncts. In many circumstances (for example when $\neg\varphi_i \equiv_\mathsf{S} \varphi_j$ for some $\varphi_i, \varphi_j \in \mathcal{F}$), these conjunctions can be simplified.

Bitstrings based on a partition

- Consider a finite fragment $\mathcal{F}$ and the partition $\Pi_{\mathsf{S}}(\mathcal{F}) = \{\alpha_1, \ldots, \alpha_n\}$ induced by it. For every $\varphi \in \mathbb{B}(\mathcal{F})$, we define a bitstring $\beta_{\mathsf{S}}^{\mathcal{F}}(\varphi) \in \{0,1\}^n$ as follows:

$$\text{for each bit position } 1 \leq i \leq n\text{: } [\beta_{\mathsf{S}}^{\mathcal{F}}(\varphi)]_i := \begin{cases} 1 & \text{if } \models_{\mathsf{S}} \alpha_i \to \varphi, \\ 0 & \text{if } \models_{\mathsf{S}} \alpha_i \to \neg\varphi. \end{cases}$$

- For each $\varphi \in \mathbb{B}(\mathcal{F})$, it holds that $\varphi \equiv_{\mathsf{S}} \bigvee\{\alpha_i \in \Pi_{\mathsf{S}}(\mathcal{F}) \mid [\beta_{\mathsf{S}}^{\mathcal{F}}(\varphi)]_i = 1\}$.
- Each formula $\varphi \in \mathbb{B}(\mathcal{F})$ can thus be written as a disjunction of anchor formulas $\alpha_i \in \Pi_{\mathsf{S}}(\mathcal{F})$, which are themselves conjunctions of (negated) formulas $\pm\varphi_j \in \mathcal{F}$ (cfr. *disjunctive normal forms*).

- if $\Pi_{\mathsf{S}}(\mathcal{F}) = \{\alpha_1, \alpha_2, \alpha_3, \alpha_4, \alpha_5\}$, and $\varphi \equiv_{\mathsf{S}} \alpha_2 \vee \alpha_3 \vee \alpha_5$, then represent φ as the bitstring 01101

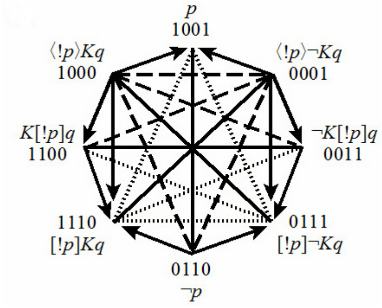
Correlation between Fragment Size and Bitstring Length

If we have a logical fragment $\mathcal{F}$ of size $m := |\mathcal{F}|$ and the partition induced by it is of size $n := |\Pi_{\mathsf{S}}(\mathcal{F})|$, then

- Theorem A bounds m in terms of n: $\lceil \log_2(n) \rceil \leq m \leq 2^n$,
- Theorem B bounds n in terms of m: $\lceil \log_2(m) \rceil \leq n \leq 2^m$.

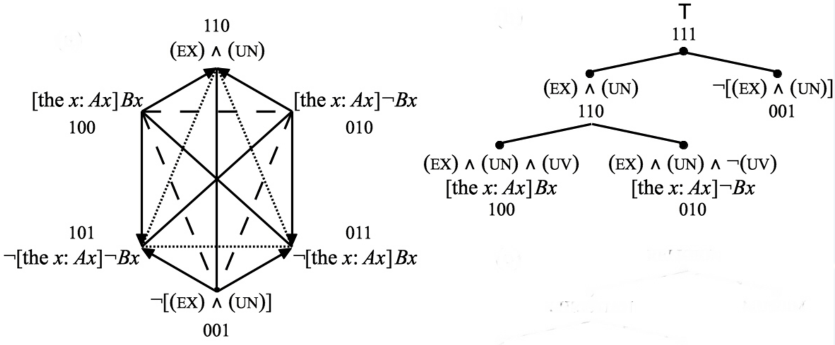
- Theorem A determines the size of a fragment m, given the minimal bitstring length n needed to represent it.
- Theorem B determines the minimal bitstring length n, given a logical fragment of size m.
- Theorems A and B can be said to be each other's inverses. The lower and upper bounds are resp. logarithmic and exponential, and thus diverge at a double-exponential rate.

Overview



For example, for the formula $K[!p]q \in \mathcal{F}$ we have:

$\models_{\mathsf{PAL}}$	$\langle !p \rangle Kq$	$\rightarrow$	$K[!p]q$	and thus	$[\beta^{\mathcal{F}}_{\mathsf{PAL}}(K[!p]q)]_1 = 1$
$\models_{\mathsf{PAL}}$	$(\neg p \wedge K[!p]q)$	$\rightarrow$	$K[!p]q$	and thus	$[\beta^{\mathcal{F}}_{\mathsf{PAL}}(K[!p]q)]_2 = 1$
$\models_{\mathsf{PAL}}$	$(\neg p \wedge \neg K[!p]q)$	$\rightarrow$	$\neg K[!p]q$	and thus	$[\beta^{\mathcal{F}}_{\mathsf{PAL}}(K[!p]q)]_3 = 0$
$\models_{\mathsf{PAL}}$	$\langle !p \rangle \neg Kq$	$\rightarrow$	$\neg K[!p]q$	and thus	$[\beta^{\mathcal{F}}_{\mathsf{PAL}}(K[!p]q)]_4 = 0$



The partition Π^{FOL}_{TDD} consists of the following anchor formulas:

$$\begin{aligned}\alpha_1 &:= [\text{the } x\colon Ax]Bx,\\ \alpha_2 &:= [\text{the } x\colon Ax]\neg Bx,\\ \alpha_3 &:= \neg[(\textsc{ex}) \wedge (\textsc{un})].\end{aligned}$$

1. Introduction
   - Central aim of Logical Geometry
   - Background of Logical Geometry
   - Structure of the tutorial
2. Bitstrings in LG: the basics
3. Decorations: applications in logic, linguistics and cognition
   - Decorations: modal logic S5
   - Decorations: subjective quantifiers
   - Decorations: gradable adjectives and colour terms
4. Bitstrings anno 2016
   - Earlier results and limitations
   - The bitstring technique
   - New decorations: public announcements, definite descriptions
5. Summary Part I

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- decorations from a wide range of fields/applications:
  - logic:
    - modal logic S5
    - Public Announcement Logic
    - definite descriptions
  - linguistics:
    - subjective quantifiers
    - proportional quantifiers
    - gradable adjectives
    - definite descriptions
  - cognition:
    - colour terms
    - knowledge representation
- new bitstring technique:
  - Boolean algebra
  - group theory
  - combinatorics

Structure of the tutorial

Thank you!

More info: www.logicalgeometry.org