LoMaDi 2025 Book of Abstracts

Leuven, Belgium 15th - 16th September, 2025

About LoMaDi

The LoMaDi workshop is a brand new workshop aiming to support the growing community of researchers who are interested in logical or mathematical diagrams, such as (but not limited to) Aristotelian diagrams, Euler diagrams, commutative diagrams, knot diagrams, Venn diagrams, Hasse diagrams, etc.

We have welcomed submissions from researchers that work directly on such kinds of diagrams, from a logical, philosophical, mathematical, historical, aesthetic, cognitive, ... perspective. Furthermore, we have also welcomed submissions from those that apply these diagrams in all sorts of research domains, be it law, psychology, computer science, linguistics, philosophy, natural sciences, or some other field. In this way, we hope to stimulate interdisciplinary work, the transfer of knowledge between fields, and the conception of creative new insights.

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Plenary Talks

On the History of How Logic Diagrams have been Applied in Mathematics

16 Sep 17:15

Jens Lemanski

Eberhard Karls University of Tübingen, FernUniversität in Hagen

The utilisation of logic diagrams has a long history. Nevertheless, there has been a lack of research into how these diagrams have actually been used in the history of logic. This is because many diagram techniques have been forgotten with the increased use of symbolic logic since the early 20th century. This lecture will focus on the application of logic diagrams in the field of mathematics. To this end, a particular focus will be placed on logic diagrams in the tradition of Lambert and Euler from the late 18th and early 19th century. These diagrams will be examined in particular, as they were used in propositional logic and applied in mathematics. The utilisation of these diagrams encompassed two primary functions: firstly as representations of individual mathematical theorems in the context of propositional logic, and secondly as substantiations or justifications within the framework of mathematical proofs. As we shall see, some of these diagrams were rediscovered in a similar form in the late 20th century, without anyone being aware of historical antecedents

Visualization and Theorematic Reasoning

15 Sep 16:35

Sun-Joo Shin Yale University

Theorems are more difficult to prove than corollaries. At the same time both are deductively inferred. How do we make a distinction between easy versus difficult deductive steps? A classification of theorems/corollaries indicates this distinction is not purely subjective but rather objective. The talk aims to show how main features of these two different kinds of reasoning are illustrated in visualization. Furthermore, I would like to link these properties to certain advantages of diagrams both as heuristics and in proofs.

16 Sep 9:00

Varieties of diagrams

Valeria Giardino CNRS-Institut Jean Nicod, Paris

In the past years, more and more work in philosophy of mathematics has been devoted to case studies on the use of visual tools in different regions of the practice of mathematics, such as Euclidean geometry, knot theory, category theory, algebra and more recently combinatorics. An exam of the discussions shows that there is no consensus on which of these tools are to be considered "diagrams". For example, knot diagrams are called as such when they seem to resemble the knot they are intended to represent, but it is not common to call a triangular figure a "diagram" of a triangle; no problem for calling a 2-dimensional object such as a matrix a diagram, while one-dimensional notations such as the formula of a circle would not always be considered as such. In spite of being only terminological issues, the questions related to what can be considered a diagram in the practice of mathematics are the occasion to offer conceptual clarifications that would help us understand what these tools are and how they are used. The aim of my talk will be to present some possible landmarks to orient ourselves in the rich territory of the variety of diagrammatic thinking in mathematics.

Contributed Talks

16 Sep 14:40

Euler-type Diagrams and Syllogistics: Operational Constraints and Notational Adequacy

Claudia Anger FernUniversität in Hagen

When working with Euler diagram systems in syllogistics, you have to deal with different operational constraints of the diagrams on the one hand and with different notation systems that accompany the diagram systems on the other. In my talk, I will first show some operational constraints, which partly arise from a different interpretation of syllogistics and can also affect the adequacy of notation systems. In addition to set theory, natural logic, and description logic, the adequacy of first order logic will be highlighted. This means: Do the respective notations also fulfill the relations in the Aristotelian diagram (square of opposition) such as (sub-)contrary, contradictory, and subaltern, which correspond to four logical connectives of propositional calculus viz., NAND, OR, XOR, and implication? Do they also fulfill the requirements of conversion, contraposition and obversion or are some of them not necessary for assertoric syllogistics? This concerns constraints such as inner and outer negation and existential import. I will also show connections to the Gergonne relations via Krause's square of opposition.

Visualising Logical Views

16 Sep 11:25

Luis F. Bartolo Alegre LMU München

In this talk, I will propose a formal characterisation of logical theories, accompanied by a visual representation of their structure.

Logical theories are often associated with formal systems – logical consequence relations defined on formal languages. However, as Hjortland (2019) observes, a logical theory encompasses more than just a formal system. This broader conception becomes especially salient when we challenge the thesis of topic-neutrality in logic – the idea that logical consequence relations must apply universally, regardless of context or subject matter. While traditionally seen as a hallmark of logic, this view has been questioned by proponents of positions such as logical contextualism and logical modalism, who argue that the appropriateness of a logical system may vary across topics or domains.

To capture this richer conception, I define a logical theory as a collection of claims structured by a triple $\mathfrak{L} = \langle \mathcal{T}, \mathcal{S}, \mathbf{C} \rangle$, where $\mathbf{C} \subseteq \mathcal{T} \times \mathcal{L}$. Here, \mathcal{T} comprises a set of topics, \mathcal{S} a set of logical systems, and \mathbf{C} a relation that indicates which logical systems are correctly applicable to which topics.

Throughout the talk, I will visually illustrate this characterisation to explore how different logical views emerge from different configurations of **C**. This visual framework aims to make explicit the interplay between logic and subject matter, and to offer a tool for better understanding the landscape of contemporary logical views.

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Revisiting Lange's Logische Studien through Beth and Hintikka: Quantification, Kantian Intuition, and Diagrammatic Inference

16 Sep 15:20

Samuel Descarreaux University of Liège, University of Lorraine

My presentation examines how the interpretations of Kantian mathematical intuition and modern quantification theory by Evert Beth and Jaakko Hintikka shed light on the largely overlooked logical contributions of the 19th-century neo-Kantian philosopher Friedrich Albert Lange (1828–1875). Lange developed an ideographical method—one that influenced thinkers such as Husserl, Peirce, and Venn—to conceptualize and intuitively represent inclusion and exclusion relationships between classes of objects. Using circular diagrams whose structural integrity remains invariant under arbitrary transformations of shape, size, or position, Lange sought to explain the validity of pure logical functions (a point that remains ambiguous in Kant), address the gap Kant left between the pure logical functions of judgment and the in concreto mathematical construction of abstract objects, and clarify the role of spatiotemporal forms of intuition in logical reasoning.

These considerations on diagrammatic logic—still relatively underexplored—become particularly illuminating when viewed through the work of Beth and Hintikka. Both authors offer partially converging interpretations of Kant's notion of mathematical construction in concreto, emphasizing its singularity while linking it differently to modern quantification theory. Their shared logical perspective highlights the Aristotelian proof by exposition (or ecthesis), traditionally associated with Euclidean constructions, and present in a conversio simplex, where a non-logical element plays a role in the construction of the theory of the syllogism.

Addressing the Locke–Berkeley problem, Beth interprets the construction of abstract mathematical demonstrations—as described by Kant, "through a chain of inferences that is always guided by intuition" (Kant [1781/7] 1980, [A716/B744], 606)—as an instance of the universal introduction rule within semantic tableaux. By introducing three individual points to form a triangle and combining them with geometric axioms and rule-based manipulations, one arrives at the universal demonstration that the sum of the angles of a triangle equals two right angles.

Rejecting Charles Parsons' phenomenalist interpretation of Kantian mathematical demonstrations, Hintikka offers a logical reading that emphasizes the role of singular representation in mathematical construction. He interprets this as the result of an existential introduction rule, which introduces a free individual symbol to replace a bound variable. This introduction—made before any reference to immediate spatiotemporal experience—explains the synthetic a priori nature of Kantian mathematical reasoning. Moreover, it reframes Kant's distinction between analytic and synthetic judgments in terms of the amount of information conveyed in a formal demonstration.

In both cases, the interpretation of in concreto mathematical construction results in an ambiguous account of the role of instantiation—either universal or existential—both of which find resonance in Lange's work. Lange not only emphasizes the role of Aristotelian proof by exposition, which he employs—like Hintikka—to

challenge the analyticity of logical reasoning, but also underscores the universality of diagrammatic demonstrations, which remain valid under arbitrary topological transformations, as Beth suggests. Clarifying the connection between instantiation rules and Kantian theories of mathematical reasoning in Lange's diagrammatic logic helps situate his contribution within the broader historical development of modern logical thought.

Between Set and Bool: Categories of Aristotelian Diagrams

Alex De Klerck KU Leuven

Aristotelian diagrams, such as the square of opposition, have long been used as pedagogical tools in logic and philosophy (Parsons, 2021). With the conception of logical geometry in the early 21st century, they became objects of study in their own right (Smessaert and Demey, 2014). Formally, a (classical) Aristotelian diagram can be viewed as a couple (\mathcal{F}, B) , where \mathcal{F} is a subset of a Boolean algebra B. When visualizing these diagrams, the Aristotelian relations (contradiction, (sub)contrariety and subalternation) that hold between the elements of \mathcal{F} are drawn. Therefore, Aristotelian diagrams exhibit two different levels of structure. The first level is defined solely by the Aristotelian relations, while the second level also cares about the other identities that hold between the elements of \mathcal{F} in B.

To establish solid mathematical foundations for the study of Aristotelian diagrams, recent research in logical geometry has incorporated category theory, by creating categories in which the objects are precisely the Aristotelian diagrams (De Klerck et al., 2024). Among them, the category $\mathbb{D}^{Inc}_{\mathcal{OR}\times\mathcal{IR}}$ looked the most promising to describe the first structural level of these diagrams.

In a forthcoming book, we deepen this category-theoretical approach to logical geometry (De Klerck et al., 2025). In this book, we show that $\mathbb{D}^{Inc}_{\mathcal{OR}\times\mathcal{IR}}$ is bicomplete by providing constructions for all its (small) limits and colimits. Additionally, we introduce the category $\mathbb{D}_{\mathcal{B}}$, which captures the second structural level of Aristotelian diagrams, and prove its bicompleteness by providing constructions for all its (small) limits and colimits. The (co)limits in these categories relate closely to those in Set and Bool, and examples of each of them can be found in previous research in logical geometry (which did not yet use the language of category theory). Furthermore, we establish three adjunctions: between Set and $\mathbb{D}^{Inc}_{\mathcal{OR}\times\mathcal{IR}}$, between $\mathbb{D}^{Inc}_{\mathcal{OR}\times\mathcal{IR}}$ and $\mathbb{D}_{\mathcal{B}}$, and between $\mathbb{D}_{\mathcal{B}}$ and Bool. Their composition is proven to be precisely the free/forgetful adjuntion between Set and Bool. Together, all of these results constitute a rich mathematical framework that exposes the core structure that governs Aristotelian diagrams.

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15 Sep 14:00

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Kinship by Diagram

15 Sep 15:20

Matt Earnshaw¹ and Thomas Holder ¹Tallinn University of Technology

Since Morgan (1870) researchers in the anthropological field of kinship studies find themselves confronted with a bewildering wealth of data and patterns. This made it necessary to experiment with various diagrammatical devices eventually resulting in a standard graphical display of kinship relations as used e.g. in (Lévi-Strauss 1945). Furthermore a distinguished tradition of "kinship algebra" came into existence which drew its concepts from the mathematical theory of groups (Weil 1949, Courrège 1965, White 1963) or generalizations thereof (Boyd-Haehl-Sailer 1972, Liu 1985) as well as formal language theory (Lounsbury 1964, Scheffler-Lounsbury 1971).

We review the development of the theory of kinship relations along the evolution of the graphical devices starting with (Macfarlane 1882) passing through the graphs used in the following of Radcliffe-Brown as well as the culmination of this tradition in the magisterial synopsis (Héran 2009).

The graphs displaying marriage systems in the anthropological literature can be identified with the Cayley graphs occurring in the group-theoretical approaches to these systems. We will discuss the potential of the kin algebra to yield further diagrammatical tools for anthropology via their modern geometrical or digrammatical conceptualisation (Munn diagrams, string calculi etc.)

As a running example we will use the kinship system of the Ambrym island that has played a prominent role in kinship studies (Rio 2005, Héran 2009) and owes its notoriety partly to a sand drawing used by the Melanesian informer to explain it (Deacon 1927). We briefly compare the development of the anthropological notations with the evolution of ideas and notations in chemistry (Mendeleev 1869, Lewis 1916) since chemical metaphors are often evoked by anthropologists (Macfarlane 1882, Lévi-Strauss 1973).

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Lost in Translation: Logical diagrams in European Vernacular Logic

16 Sep 16:35

Jasper Eeckhout KU Leuven

In the sixteenth century the discipline of logic went through a period of extensive change. Not only did a new humanist tradition of teaching and practising logic develop next to a declining scholastic tradition, but for the first time, authors from a multitude of different backgrounds attempted to write logic textbooks in their own vernacular as opposed to the Latin of the scholastics. In the last decades, scholarship has addressed the development of logic in some (but not all) of the different European vernaculars. However, up until this point, virtually no attempt has been made to study the patterns and tendencies that hold across the different vernaculars in which these logic textbooks are written. One salient starting point to address these cross-vernacular patterns are the logical diagrams that are presented in these textbooks. More specifically, the Aristotelian diagrams of the first vernacular textbooks exhibit essential logical vocabulary as well as significant traces of the Latin textbooks the authors used.

My work aims to show that, and to investigate how, the Aristotelian diagrams of the first vernacular textbooks give key insight into some of the patterns, reasons and causes behind the transition from Latin to the different European vernaculars for the teaching and practise of logic. Firstly, with the help of Aristotelian diagrams, I show that the majority of the new logical vocabulary is the result of more conservative translation techniques (e.g., borrowing, calque) as opposed to more original/creative techniques (e.g., metaphors). Furthermore, I argue that the choice between a conservative and a more original translation style corresponds with a difference in motivation of the authors to write in the vernacular. In addition, using Aristotelian diagrams, I demonstrate how the sources these authors used in their own logical project show the indebtedness of the vernacular transition to the humanist tradition of logic textbooks.

The purpose of my work is to broaden our understanding of the transition from Latin to the different European vernaculars with regard to the teaching and practice of logic. In addition, my work endeavours to augment our account of the use of logical diagrams throughout history. Finally, my work aims to expand our view on the history of the democratisation of knowledge and on an important chapter in the development of the European vernaculars as languages of science.

A Diagrammatic Way of Finding Minimal and Maximal Boolean Complexities

15 Sep 10:15

Atahan Erbas KU Leuven

Finding the minimal and the maximal Boolean complexities of a given family of Aristotelian diagrams (ADs) is one of the key issues in logical geometry (Demey, 2018). For example, the minimal and maximal Boolean complexities of the families of JSB hexagons (Demey, 2018), U4 hexagons (Demey and Erbas, 2023), Buridan octagons (Demey, 2019) and KJ octagons (Demey and Smessaert, 2024) were found in the literature. While Demey (2018) offers a systematic way of finding the maximal complexity of a given family of ADs, none of these works provides a systematic one for finding the minimal Boolean complexity of a given family of ADs. In other words, these works offer various ways of finding the minimal Boolean complexity of a given family of ADs but each depends on a particular case of a family of ADs. This paper offers a systematic and diagrammatic way of finding the minimal and the maximal Boolean complexities of a given family of Aristotelian diagrams. This diagrammatic way exemplifies the non-redundant role of diagrams in logical proofs and a case for rigor pluralism in logic (De Toffoli, 2022; Tanswell, 2024). The paper proceeds as follows. To set the stage, Section 1 introduces the concepts of families and subfamilies within the typology of Aristotelian diagrams. After this gentle introduction, Section 2 argues that the ways of finding the minimal Boolean complexity of a given family of ADs found in the literature are not contextindependent enough to be systematic. Inspired by the argumentation frameworks used in the field of logical argumentation, Section 3 constructs new diagrams that we call anchor diagrams. Anchor diagrams are Aristotelian diagrams in disguise and will be used in finding the minimal and the maximal Boolean complexities of a given family of ADs. Looking through the lens of the philosophy of logical practice, Section 4 argues that anchor diagrams play a non-redundant role in proofs within logical geometry. Finally, Section 5 discusses some side benefits of anchor diagrams for the typology of ADs, which will be realized in a future work.

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15 Sep 12:05

Aristotelian Diagrams and Many Valued Logics

Stef Frijters KU Leuven

The square of opposition and other, more complex, Aristotelian diagrams have been used to graphically represent logical relations since at least the early middle ages. Recently there has been a notable surge in the study of these diagrams as objects of independent interest, often under the name 'logical geometry'. Most, though certainly not all, of the field of logical geometry has however only been concerned with diagrams based on classical logics. In this talk I will focus on the logical geometry of non-classical, specifically many-valued, logics.

The relations expressed by Aristotelian diagrams are usually defined in terms of truth values. For example, two formulas are said to be contrary to each other iff they cannot be *true* together, but can be *false* together. Instead, I propose a slight generalization of these definitions in terms of *designated values*. For example, two formulas are contrary to each other iff they cannot be *designated* together, but can be *undesignated* together.

I show that this is a proper generalization, i.e. any pair of formulas of a classical logic is in one of the *-relations iff they are in the corresponding 'normal' relation. However, the new definition also allows us to study the Aristotelian Diagrams for, and the logical geometry of many-valued logic. This is what I will do in this talk.

Take for example the set $\{p \land q, p \lor q, \neg p \land \neg q, \neg p \lor \neg q\}$. It is well-known that in classical logic this gives us a square of opposition. If instead we draw the Aristotelian diagram for the same formulas in Kleene's three-valued 'strong logic of indeterminacy' (using the * definition), then we get a square of opposition where the subcontrariety relation is missing. In contrast, if we draw the diagram for Priest's three-valued 'logic of paradox', then the contrariety relation is missing. This shows that there is both a discontinuity and a continuity between the Aristotelian geometry of classical and many-valued logic. The discontinuity is illustrated by the fact that the two new diagrams were impossible to draw in classical logic (i.e. there is no set of four formulas such that they give an Aristotelian isomorphic diagram in classical logic). The continuity lies in the logic-sensitivity of the new diagrams: in the classical case, changing underlying logics gives different diagrams. This example shows that the same holds for many-valued logics.

Aristotelian Geometry and Weak Trivalent Logic

16 Sep 12:05

Jan Heylen and Stef Frijters KU Leuven

Our starting point is Aristotelian geometry and Aristotelian diagrams, as defined by Smessaert and Demey (2014). We want to apply this to systems of weak trivalent logic, in which the semantical values of sentences are truth, falsity or neutrality, and in which the semantical values of negations, conjunctions, disjunctions and conditionals are determined by the weak Kleene truth tables. When it comes to the semantical values of quantified sentences, there are choices to be made, resulting in multiple systems of weak trivalent first-order logic. When it comes to the semantical notion of validity, there are also choices to be made, which is an additional reason why there are multiple systems of weak trivalent logic.

We suggest a new approach to Aristotelian geometry in which not the notion of logical truth but rather the notion of validity is central. There is both a general reason and a specific reason for this. The general reason is that it makes sense to use a relational notion, namely logical implication, to capture other relational notions, such as the Aristotelian relations (contradiction, contrariety, subcontrariety, subalternation) and pairwise non-equivalent formulas. The specific reason is that the system of weak trivalent logic does not have any logical truths, whereas it does have valid arguments. If there are no logical truths in a logical system, then the four Aristotelian relations for that system are each identical to the empty set and the Aristotelian geometry of the system is the singleton containing the empty set. Moreover, in that case all sentences are pairwise non-equivalent.

In our new approach to Aristotelian geometry we will carefully examine which notions of validity are conceptually the most interesting ones to work with. The outcome is a non-trivial Aristotelian geometry for systems of weak trivalent logic. The Aristotelian diagrams differ from the ones hitherto studied in the literature.

16 Sep 14:00

Finding Aristotle's "lost" syllogistic diagrams

Zoe McConaughey Université de Lille

Diagrams often accompanied Aristotle's Analytics in various traditions from the 5th century CE onward. Some commentators doubt that Aristotle himself resorted to such lune-and-triangle diagrams. For instance, Reviel Netz (2022, p. 11) claims that "modern scholars know full well that this scholiastic tradition begins not earlier than Late Antiquity." On the contrary, Michel Crubellier (2014, p. 381) suggests that the lune-and-triangle diagrams found in the margins of manuscripts were actually Aristotle's own diagrams, though he does not give substantial evidence for this claim.

This talk intends to provide textual evidence in favor of Crubellier's hypothesis. Instead of looking at the beginning of the Prior Analytics, where Aristotle systematically studies the three syllogistic figures, I will argue that the genesis of the lune-and-triangle diagrams is to be found in the middle of the Prior Analytics, in the section called the "pons asinorum" by scholastics (APr. I 26-31). I will show how the diagrams can be built (1) out of a basic spatial structure described in chap. 27 and 28, and (2) through the manipulation of this structure in chap. 28.

Arguing that the manuscript diagrams were Aristotle's own pursues two linked goals. First, it brings attention to these diagrams: far from being non-Aristotelian, the diagrams found in the margins of manuscripts are actually Aristotle's own diagrams, which would therefore never have been lost. Second, it paves the way to a renewed and diagrammatical reading of Aristotle's Prior Analytics: understanding the diagrams is a key to understanding how the three figures are related, how a reduction to impossibility works, how it differs from a reduction to a first figure and how it relates to it.

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Diagrammatic Experiments and Mathematical Creativity: Integrating the Frameworks of Margaret Boden and Charles S. Peirce to Clarify the Mechanisms of Mathematical Invention

15 Sep 14:40

Matías Saracho Universidad Nacional de Córdoba

How is mathematical knowledge produced? While the philosophy of mathematical practice today grapples with this long-standing question, and substantial progress has been made in studying mathematical creativity and reasoning, a comprehensive framework that fully captures the complexities of the cognitive processes involved in mathematical invention is still lacking. To contribute to such a framework, our project explores the integration of Margaret Boden's typology of creativity and Charles Sanders Peirce's philosophy of mathematics, particularly his insights into the nature of mathematical reasoning and discovery.

Many of Peirce's main contributions derived from his attempt to understand how necessary reasoning can produce novelties in mathematics, a problem that puzzled him throughout his entire life. This drive motivated his keen interest in logic, ultimately leading him to wonder whether all mathematical reasoning is diagrammatic, and subsequently to distinguish between two kinds of deductive reasoning: corollarial and theorematic. The former resembles Kant's notion of deductive reasoning and consists in seeing the solution to a problem directly within the diagram of the premises. The latter is much more complex and requires ingenuity. It demands experimenting on the diagram of the premises to uncover hidden relations before the solution to the problem can be observed. For the purpose of the experiment, the diagram becomes a tangible object of inquiry, subject to observation and manipulation in unpredictable ways ((Peirce, 1976), NEM 4:38). Peirce's notion of theorematic deduction has attracted significant contemporary attention in relation to current research on mathematical practice (e.g., (Carter, 2010)). Nonetheless, the underdeveloped nature of the notion is acknowledged, and further clarification is deemed necessary (Stjernfelt, 2013).

For her part, Boden is interested in creativity as a basic human capacity which she defines as the capacity to come about with ideas that are new, surprising, and valuable. This leads her to identify three sources, and consequently three types, of creativity: the combination of familiar ideas in an unexpected way or combinational creativity, the exploration of conceptual spaces or exploratory creativity, and the transformation of those spaces or transformational creativity (Boden, 2004). Therefore, her theory offers a vantage point through which to clarify Peirce's concept of theorematic deduction. Mapping Boden's typology onto Peirce's descriptions of theorematic reasoning demonstrates that Peirce's notion involves the very three types of creativity: combinational, exploratory, and transformational creativity. Conversely, Peirce's detailed account of diagrammatic reasoning provides a cognitive grounding for Boden's high-level theory. In effect, by conducting experiments on diagrams, mathematicians discover novel, surprising, and valuable connections, explore, and transform conceptual spaces.

Through an examination of historical case studies in mathematics, including Desargues Theorem, Euclid's Pons Asinorum, the emergence of non-Euclidean geometry, and the development of calculus, we seek to demonstrate how this complementary approach can illuminate diverse process underlying mathematical innovation. In our presentation, we will outline the early stages of this work and conclude by discussing the strengths and limitations of this integrated framework, its implications for understanding how mathematicians create new knowledge, and potential directions for future research in this area.

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Visual Reasoning and Existential Import: a Historical and Comparative Study

16 Sep 10:15

Karol Wapniarski and Mariusz Urbański Adam Mickiewicz Univeristy

The study of logical diagrams and their role throughout the history of logic is recently getting more and more attention from scholars (e.g. (Bhattacharjee, 2024), (Hodges, 2023)). This talk aims to present a particular case study from history of syllogistic logic to illustrate the influence that a use of particular diagrammatic representations may have on philosophical views on logic. Specifically, we look into how logical diagrams used historically to render syllogistic moods might have influenced the debate on the use of empty terms in logic, and, consequently, how the evolution of those diagrams have affected the parallel evolution of views on empty terms.

Since the problem of existential import was treated differently throughout the centuries (arguably absent in Aristotle, "discovered" by the Arabic logicians, and in Europe debated mostly by the nominalists (Wapniarski et al., 2024)), we propose to look at the parallel evolution of logical diagrams in order to better understand these shifts in attention. To this end, we consider the Byzatine diagrams for syllogistic figures, linear diagrams of Al-Barakāt, Leibniz's and Lambert's linear diagrams, Euler circles, Venn, and Venn-Peirce diagrams. We draw particular attention to how the (in)ability of the diagrams to account for empty terms corresponds with the (ir)recognition of the problem of admitting empty terms throughout history.

By discussing Euler circles, Venn, and Venn-Peirce diagrams, we also aim to show the influence Modern Formal Logic started to exert on the understanding of Aristotelian Syllogistic at that time. Effectively, the talk is an attempt at extending the perspective on the interconnections between visual reasoning and recognizing existential commitments of logic presented in (Lemanski, 2024) to encompass the evolution of logical diagrams also before the nineteenth century.

References

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11:25

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Diagrams of word meaning exist in many different forms and contexts, in anthropology, artificial intelligence, cognitive science, linguistics, logic. A square of opposition can show how quantificational words (every, some, no) relate logically to each other (Horn, 1989), a tree can represent the structure of ethnobiological vocabulary (Kay, 1971), a Venn diagram how word meanings overlap (Geeraerts et al., 2012), and a plot their fuzzy boundaries (Labov, 1973). We find various types of 'semantic maps' in cross-linguistic typology (Haspelmath, 2003) and 'conceptual spaces' in all sorts of domains (Gärdenfors, 2000) while the development of distributional semantics is leading to a rich spectrum of diagrams, showing changes in word meaning, for instances (Hamilton et al. 2016).

Although there is some general work on diagrams in linguistics (Hamilton et al., 2016; Smessaert and Demey, 2018; Bubenhofer, 2020), we still lack the bigger picture on the different ways in which word meaning is (or can be) visualized and we lack answers to important questions. What kind of graphic object is a word meaning? Does every type of diagram visualize word meanings in its own specific way? What kind of word meaning diagrams are possible?

We answer these questions by a two-dimensional typology of word meaning diagrams (applied to a corpus of $\pm 2K$ examples). One dimension is graphic type. Building on (Richards and Engelhardt, 2020) we extend the fundamental triad of pictorial, textual and geometric components and distinguish three broad types of composite graphics: pictures, texts and diagrams, the latter further divided into plots, tables, networks (incl. trees), set diagrams and clouds (depending on their geometric strategies). The other dimension is word shape: graphically, word meanings can be points, arrows/lines or regions inside a graphic or the graphic as a whole (space). This gives us 28 theoretical possibilities, almost all of which instantiated in the corpus.

	text	table	cloud	set diagram	network	picture	plot
point	₹		•	(D)	>	*	<u> </u>
line	=	*	#	(3)	>	*	<u> </u>
region	=		**	0		**	$\overline{\wp}$
space	≣	\blacksquare	#	0	Y	*	L

The word every in a square of opposition is a point in a network, for instance, but semantic maps (Haspelmath, 2003) and frames (Löbner, 2013) are also networks, although with different shapes (region and whole space, respectively). Also, one and the same word, like cup, can be represented as a point in a table (Lehrer, 1974), a line in a plot (Labov, 1973), a region in a cloud (Malt et al., 2003), a whole set diagram (Löbner, 2013).

In this way, by combining what we know about general graphical possibilities with specific approaches to word meaning (intensional vs. extensional, onomasiological vs. semasiological), we get insight in the expressive power of the visual metalanguages that are important for the many disciplines studying words, concepts and categories.

References

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Labov 1973 The boundaries of words and their meanings

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Hamilton et al. 2016 Diachronic word embeddings reveal statistical laws of semantic change

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Smessaert and Demey 2018 Towards a typology of diagrams in linguistics

Bubenhofer 2020 Visuelle Linguistik

Richards and Engelhardt (2020) The DNA of information design for charts and diagrams

Löbner 2013 Understanding Semantics

Lehrer 1974 Semantic Fields and Lexical Structure

Malt et al. 2003 Universality and language specificity in object naming