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Shape Heuristics in Aristotelian Diagrams

Lorenz Demey and Hans Smessaert

Shapes 3.0 Workshop, Larnaca, 2 November 2015

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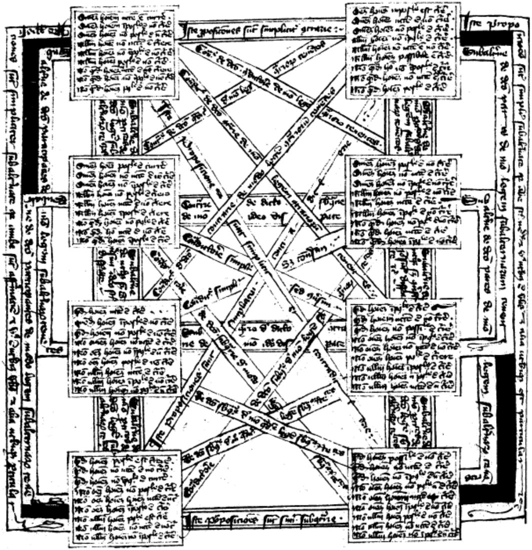
- Aristotelian diagram
  - compact visual representation
  - of the elements of some logical/lexical/conceptual field
  - and the logical relations holding between them

- most widely known example: square of oppositions

- intellectual background
  - rich history in philosophical logic
    - starting in the 2nd century AD (Apuleius)
    - especially popular in medieval logic
  - today: used in various disciplines
    - cognitive science, linguistics, law...
    - computer science, neuroscience...

⇒ Aristotelian diagrams as a *lingua franca* for an interdisciplinary research community concerned with logical reasoning


Purpuree
Iliace
Contrarie
Subcontrarie
Amabim?
Eventuli
Incipit Secundus Tractatus

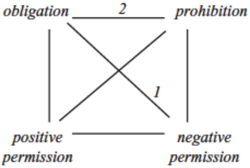


The European Journal of International Law Vol. 17 no.2

The Definition of 'Norm Conflict' in International Law and Legal Theory

Erich Vranes*

The possible set of inter-relations can be illustrated by using the so-called deontic square, which in fact relies on the logic square known since Greek antiquity,[85] and which was arguably first used in deontic logic by Bentham:[86]



Universal vs. particular reasoning: a study with neuroimaging techniques

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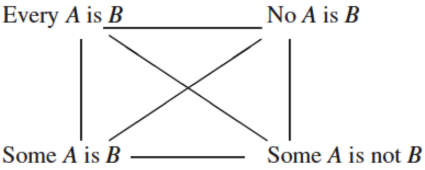
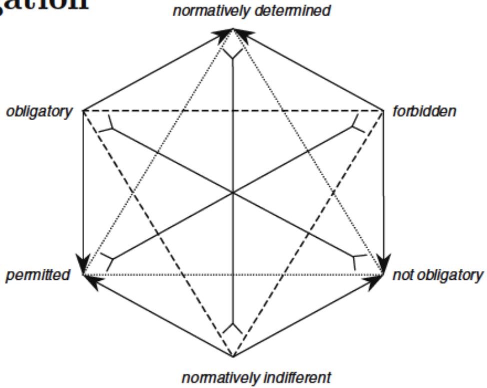


Deontological Square, Hexagon, and Decagon: A Deontic Framework for Supererogation

Jan C. Joerden



- research project: logical geometry
- study **new decorations** of Aristotelian diagrams
  - historical case studies (e.g. Avicenna, Ockham, Keynes)
  - applications in various fields (e.g. philosophy of language, AI)
- study Aristotelian diagrams as **objects of independent interest**
  - abstract-logical aspects: information, context-sensitivity, etc.
  - visual-geometrical aspects: dimension, perpendicularity, collinearity, etc.

  $\Rightarrow$ shape characteristics of Aristotelian diagrams!

- aim of this talk: argue that Aristotelian diagrams' shape can have great heuristic value (based on earlier 'geometric' work in logical geometry)

- the Aristotelian relations: given logical system S, formulas φ and ψ are

contradictory	iff	$S \models \neg(\varphi \wedge \psi)$	and	$S \models \neg(\neg\varphi \wedge \neg\psi)$
contrary	iff	$S \models \neg(\varphi \wedge \psi)$	and	$S \not\models \neg(\neg\varphi \wedge \neg\psi)$
subcontrary	iff	$S \not\models \neg(\varphi \wedge \psi)$	and	$S \models \neg(\neg\varphi \wedge \neg\psi)$
in **subalternation**	iff	$S \models \varphi \rightarrow \psi$	and	$S \not\models \psi \rightarrow \varphi$

- informal explanation:
  - contradiction, (sub)contrariety: formulas can(not) be true/false together
  - subalternation: one-way logical entailment

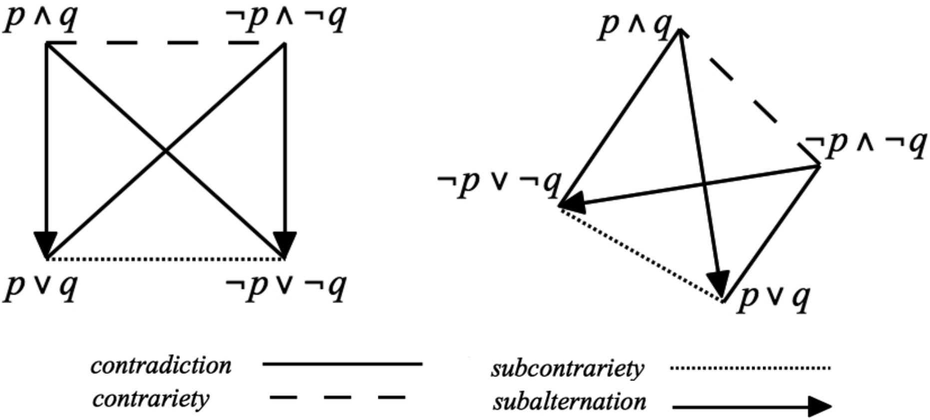
- Aristotelian diagrams only contain contingent formulas
  - tautology ($\top$) and contradiction ($\bot$) are not present in the diagram
  - alternative view: $\top$ and $\bot$ coincide in the center of the diagram ($\Rightarrow$ not a separate vertex) (Sauriol in the 1950s, Smessaert in the early 2000s)

- Aristotelian diagram crucially depends on formulas and logical system S
  - different formulas ⇒ different diagram
  - different logical system ⇒ different diagram
- suppose that formulas and logical system have been fixed
  - logical properties of the diagram are fully determined
  - visual-geometric properties still seriously underspecified
    ⇒ various design choices possible
- multiple diagrams for the same formulas and logical system
  - informationally equivalent: contain the same logical information
  - not necessarily computationally/cognitively equivalent: one diagram might be more helpful/useful than the other ones
    (ease of access to the information contained in the diagram)

Jill Larkin and Herbert Simon, 1987
Why a Diagram is (Sometimes) Worth 10.000 Words

Example: Square (2D) versus Tetrahedron (3D)

- four formulas: $p \wedge q, p \vee q, \neg p \wedge \neg q, \neg p \vee \neg q$
- logical system: classical propositional logic (CPL)



The Heuristic Value of Shape

- how to choose among informationally equivalent diagrams?
- shape can have powerful heuristic function

- consider the set(s) of formulas represented by Aristotelian diagram(s)
- these have various properties and relations amongst each other

- in good (cognitively helpful) Aristotelian diagrams, the diagrams' shape helps to visualize these properties and relations

[abstract-logical]		[visual-geometric]
properties, relations among sets of formulas	⟵ isomorphism ⟶ congruity	shape characteristics of the diagrams

Corin Gurr, Barbara Tversky

- isomorphism between
  - abstract-logical subject matter
  - visual-geometric (shape) properties

- good diagram simultaneously engages the user's logical and visual cognitive systems

  ⇒ facilitate inferential or heuristic **free rides** (Atsushi Shimojima)

  - logical properties are directly manifested in the diagram's visual features
  - user can grasp these properties with little cognitive effort

"you don't have to reason about it, you just see it"

- how to choose between informationally equivalent Aristotelian diagrams D1 and D2?

- informationally equivalent ⇒ same logical subject matter
- differerent shapes
  - shape of D1 more clearly isomorphic to subject matter
  - shape of D2 less clearly isomorphic to subject matter

- D1 will trigger more heuristics than D2
- ceteris paribus, D1 will be a more effective visualization than D2

  ⇒ D1 and D2 are not computationally/cognitively equivalent

- remainder of the talk: two (series of) case studies
  - Aristotelian diagrams for entire Boolean algebras
  - complementarities between Aristotelian diagrams

Boolean Closed Aristotelian Diagrams

- Aristotelian diagrams that are Boolean closed
- Aristotelian diagrams for entire Boolean algebras (except for $\top$, $\bot$)

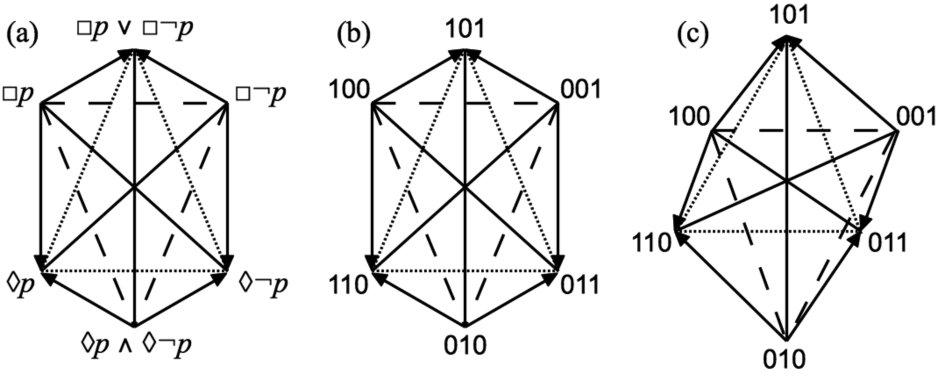
- finite Boolean algebra $\Rightarrow$ bitstring representation

- first interesting case: Boolean algebra $\mathbb{B}_3$ (bitstrings of length 3)
  - in total $2^3 = 8$ formulas/bitstrings
  - after leaving out $\top$ and $\bot$ (i.e. 111 and 000): 6 formulas/bitstrings

- Jacoby-Sesmat-Blanché (JSB) diagram
  - most common visualization: hexagon (2D)
  - alternative visualization: octahedron (3D)

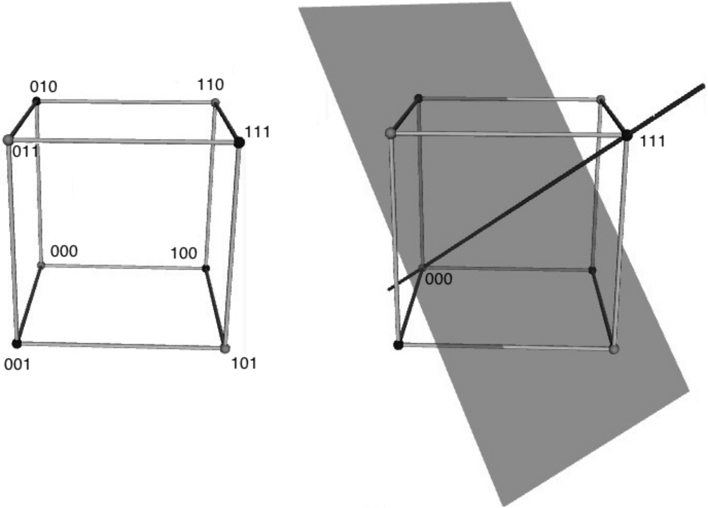
$\Rightarrow$ informationally equivalent,
but also computationally equivalent?

JSB: Hexagon (2D) vs. Octahedron (3D)



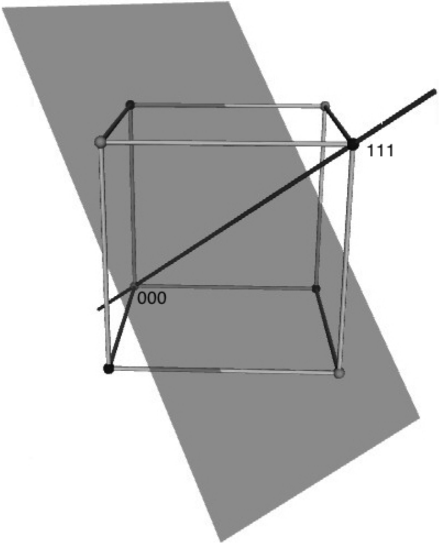
- Boolean algebra $\mathbb{B}_n$ (bitstrings of length n)
  - can be represented as n-dimensional hypercube ("Boolean cube")
  - bitstrings not only as logical entities,
    but also as coordinates of vertices in n-dimensional space
- in case $n = 3$, we have an 'ordinary' cube (3D)
- vertex-first projection of this cube along the 111/000 axis

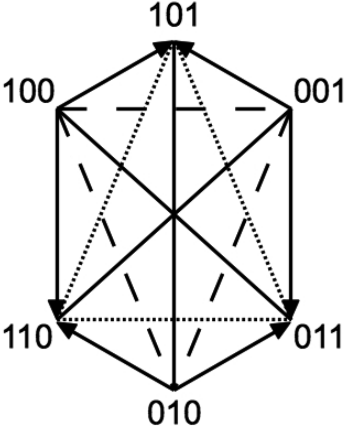
  $\Rightarrow$ result: JSB hexagon



010
110
111
011
000
100
001
101
111
000

Vertex-First Projection of the Cube





- JSB hexagon ⇜⇝ (projection of) Boolean cube
- projection axis is defined by the non-contingent bitstrings 111/000
  - 111 and 000 not part of the hexagon
  - 111 and 000 coincide in the center of the hexagon

[abstract-logical]		[visual-geometric]
Boolean closed	⟵ isomorphism ⟶	hexagonal shape
only contingent bitstrings		111/000 coincide in the middle

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- two common types of diagrams for $\mathbb{B}_3$
  - Aristotelian diagram (JSB) (hexagon, octahedron. . .)
  - Hasse diagram (hexagon, . . .)

  - Aristotelian hexagon = projection of cube along 111/000 axis
  - Hasse hexagon = projection of cube along any other axis (e.g. 101/010)

- strong connection between Aristotelian and Hasse diagram for $\mathbb{B}_3$
  $\Rightarrow$ unified explanation for their similarities and differences

- hexagonal JSB diagram for $\mathbb{B}_3$ has several cognitive advantages
  (octahedral JSB diagram for $\mathbb{B}_3$ lacks these advantages)

- hexagonal and octahedral JSB diagram for $\mathbb{B}_3$
  - informationally equivalent
  - certainly not computationally equivalent

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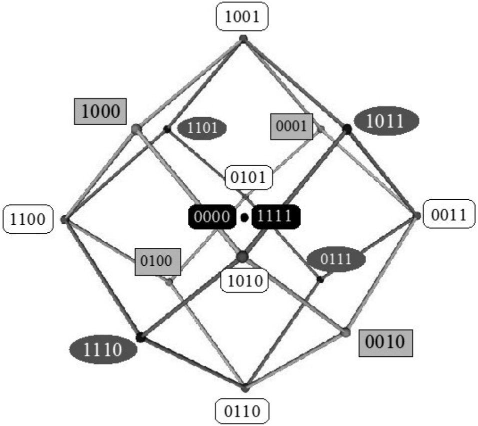
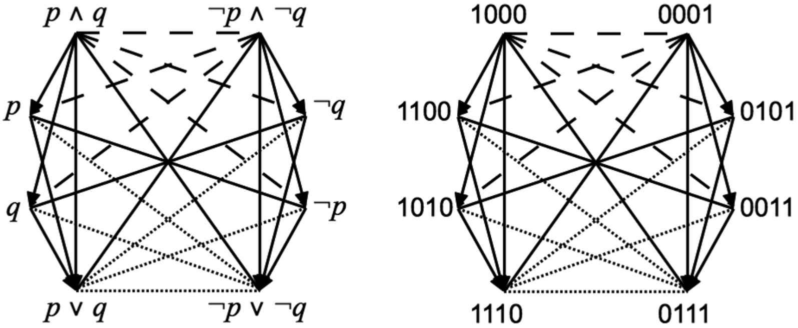
- analogous story
  - various Aristotelian diagrams for $\mathbb{B}_4$
  - best one: rhombic dodecahedron (RDH) = projection of 4D hypercube



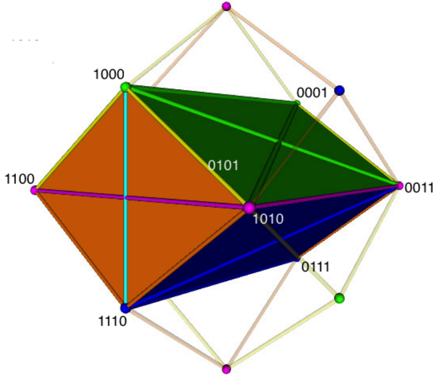
Buridan Diagram for Propositional Logic

- Buridan diagram = widely studied type of Aristotelian diagram
- example: Buridan diagram for propositional logic
- can be represented by bitstrings of length 4



Buridan Diagram: Octagon (2D) vs. Rhombicube (3D)

- Buridan diagram: usually visualized by means of an **octagon**
- representable by bitstrings of length 4
  $\Rightarrow$ subdiagram of the RDH for $\mathbb{B}_4$: **rhombicube**
- informationally equivalent, but also computationally equivalent?



- rhombicube: level $\longleftarrow$isomorphism$\longrightarrow$ verticality
  - level-1 bitstrings (1000, 0001) at the top of the diagram
  - level-2 bitstrings (1100, 1010, 0101, 0011) in the middle
  - level-3 bitstrings (1110, 0111) at the bottom of the diagram

$\Rightarrow$ cannot be achieved in a 2D octagon visualization

- rhombicube = subdiagram of RDH (shared rhombic faces)
  - via its shape, the rhombicube establishes a link with RDH ($\mathbb{B}_4$)
  - suggests that it can be represented by bitstrings of length 4

- rhombicube stands in geometric complementarity with hexagon

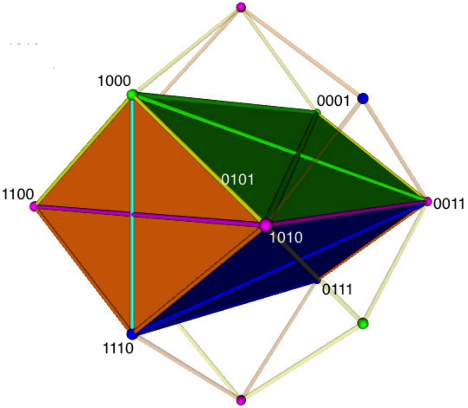
$\Rightarrow$ reflects an underlying logical complementarity
between Buridan and JSB diagrams

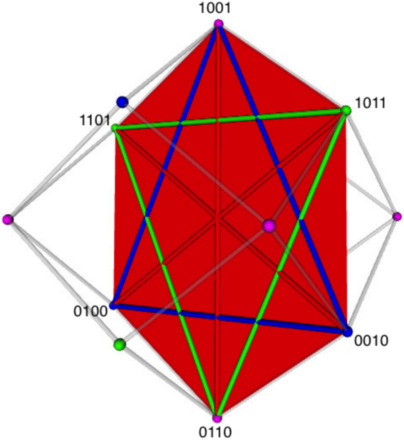
- logical complementarity between Buridan diagram and JSB diagram
  - $\mathbb{B}_4$ has 16 bitstrings (14 after excluding 1111 and 0000)
  - 8 bitstrings have $\neq$ values in bit positions 1 and 4 $\Rightarrow$ Buridan diagram
  - 8 bitstrings have $=$ values in bit positions 1 and 4;
    6 after excluding 1111 and 0000 $\Rightarrow$ JSB diagram

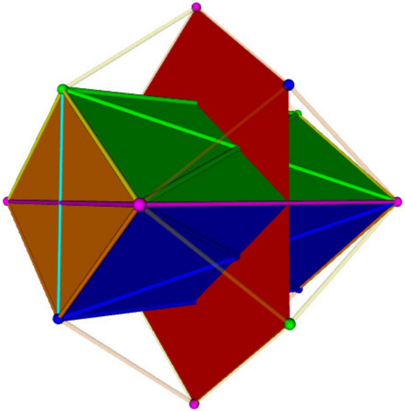
1000	0111	1001	0110
0001	1110	1101	0010
1100	0011	1011	0100
0101	1010	(0000)	(1111)

- geometric complementarity between rhombicube and hexagon
  - Buridan embedded inside RDH: rhombicube $\Rightarrow$ partition
  - JSB embedded inside RDH: hexagon of RDH
- rhombicube visualization of Buridan diagram
  - geometric complementarity with JSB hexagon
  - reminder of underlying logical complementarity

Logico-Geometrical Complementarity: Rhombicube/Hexagon







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- several diagrams for a given set of formulas and logical system:
  - informationally equivalent, but not always computationally equivalent
  - diagrams' shape can play a heuristic role
- two (series of) case studies (building on earlier work):
  - Aristotelian diagrams for entire Boolean algebras
  - complementarities between Aristotelian diagrams
- future work: investigate the heuristic role of shape in Aristotelian diagrams that are not covered by the present series of case studies (e.g. how to visualize a Sherwood-Czezowski diagram?)

Thank you!

More info: www.logicalgeometry.org