Duality and Lexicalization in Medieval Squares of Opposition

Lorenz Demey

3rd CLAW/DWMC Symposium, 31 May 2017
two issues related to the square of opposition
  • Aristotelian vs. duality relations
  • (non-)lexicalization

each of them **separately** is (relatively) well-understood

this talk: explore the **interaction** between these two issues
  • argue that they mutually reinforce each other
  • use this interaction to shed new light on some issues in medieval logic

based on joint work with Hans Smessaert and Dany Jaspers
Structure of the talk

1. Aristotelian Relations and Duality Relations
2. Lexicalization in Aristotelian Diagrams
3. The Interaction between Duality and Lexicalization
4. Duality and Lexicalization in Medieval Logic
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an Aristotelian diagram visualizes some formulas/expressions and the Aristotelian relations holding between them

two propositions are said to be

contradictory iff they cannot be true together and they cannot be false together,

contrary iff they cannot be true together but they can be false together,

subcontrary iff they can be true together but they cannot be false together,

in subalternation iff the first proposition entails the second but the second doesn’t entail the first
Vowel convention for the square of opposition
Example 1: the quantifier square

(assumption of existential import: there exists at least one S)
Example 2: the modal square
The duality relations

- many Aristotelian diagrams not only exhibit Aristotelian relations, but also duality relations among their elements

- view a proposition $\varphi$ as the output of some $n$-ary operator $O$ on some inputs $x_1, \ldots, x_n$: $\varphi = O(x_1, \ldots, x_n)$

- given two operators $O_1, O_2$, we say that
  
  $O_2$ is the **internal negation** of $O_1$  
  iff $O_2(x_1, \ldots, x_n) \equiv O_1(\neg x_1, \ldots, \neg x_n)$  

  $O_2$ is the **external negation** of $O_1$  
  iff $O_2(x_1, \ldots, x_n) \equiv \neg O_1(x_1, \ldots, x_n)$

  $O_2$ is the **dual** of $O_1$  
  iff $O_2(x_1, \ldots, x_n) \equiv \neg O_1(\neg x_1, \ldots, \neg x_n)$
Example 1: the quantifier square

\[
\begin{array}{ccc}
\text{all } S \text{ are } P & \text{no } S \text{ are } P \\
\text{some } S \text{ are } P & \text{some } S \text{ are not } P
\end{array}
\]

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Example 2: the modal square
D: defined for formulas of the form $\varphi = O(x_1, \ldots, x_n)$
A: defined for all formulas

D: **symmetric**: if $R(\varphi, \psi)$ then $R(\psi, \varphi)$
A: subalternation is antisymmetric: if $SA(\varphi, \psi)$ then not $SA(\psi, \varphi)$

D: **deterministic**: if $R(\varphi, \psi_1)$ and $R(\varphi, \psi_2)$ then $\psi_1 \equiv \psi_2$
A: a formula can have **multiple** contraries

D: **serial**: for all $\varphi = O(x_1, \ldots, x_n)$, there exists $\psi$ such that $R(\varphi, \psi)$
A: a formula can have **no** contraries at all

D: four by four: $\{O, \text{INEG}(O), \text{ENEG}(O), \text{DUAL}(O)\}$ (Klein 4-group)
A: squares, but also hexagons, octagons, etc.

D: not sensitive to the details of the underlying logical system $S$
A: highly **logic-sensitive**: contradictories in $S_1$, contraries in $S_2$
- Jacoby-Sesmat-Blanché (JSB) hexagon
- Boolean closure of the square
Logic-sensitivity: the modal square in KD versus K

Duality and Lexicalization in the Square – L. Demey
conceptual independence of Aristotelian and duality relations

nevertheless: many (all?) squares in the philosophical/logical literature are simultaneously Aristotelian squares and duality squares

- classical examples (cf. middle ages): quantifiers, modalities
- contemporary examples: definite descriptions, public announcement logic
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3. The Interaction between Duality and Lexicalization

4. Duality and Lexicalization in Medieval Logic
Example 1: the quantifier square

- the A-corner is primitively lexicalized as *all*
- the I-corner is primitively lexicalized as *some*
- the E-corner is primitively lexicalized as *no*
- the O-corner is not primitively lexicalized
Example 2: the modal square

- the A-corner is primitively lexicalized as *necessary*
- the I-corner is primitively lexicalized as *possible*
- the E-corner is primitively lexicalized as *impossible*
- the O-corner is not primitively lexicalized
not just with quantifiers and modalities, but also in **other lexical domains**

- all, some, no vs. some not
- necessary, possible, impossible vs. possible not
- everywhere, somewhere, nowhere vs. somewhere not
- everybody, somebody, nobody vs. somebody not
- always, sometimes, never vs. sometimes not
- both, either, neither vs. either not

not just in English, but also in **other natural languages**

first author to point this out: Thomas Aquinas, *In Arist. De Int. (Expositio libri Peryermeneias)*, Book I, Lesson 10
Sicut autem supra dictum est, quandoque aliquid attribuitur universali ratione ipsius naturae universalis; et ideo hoc dicitur praedicari de eo universaliter, quia scilicet ei convenit secundum totam multitudinem in qua invenitur; et ad hoc designandum in affirmativis praedicationibus adinventa est haec dictio, omnis [...] In negativis autem praedicationibus adinventa est haec dictio, nullus [...] 

Quandoque autem attribuitur universali aliquid vel removetur ab eo ratione particularis; et ad hoc designandum, in affirmativis quidem adinventa est haec dictio, aliquis vel quidam, per quam designatur quod praedicatum attribuitur subiecto universali ratione ipsius particularis; sed quia non determinate significat formam alicuius singularis, sub quadam indeterminatione singulare designat; unde et dicitur individuum vagum. In negativis autem non est aliqua dictio posita, sed possumus accipere, non omnis
systematic explanation of the non-lexicalization of the O-corner
Horn: pragmatic (Gricean) account
Jaspers: JSB hexagon = square + Y-corner (below), U-corner (above)
  the Y-corner is (often) co-lexicalized with the I-corner
  the U-corner is not lexicalized
Seuren and Jaspers’ account

- recursive partitioning of the universe
- not lexicalized: disjunction **across** subuniverse
  - quantifier U-corner: *all or no*
  - modal U-corner: *necessary or impossible*
  - quantifier O-corner: *some*$_1$ *not* $\equiv$ *some*$_2$ *or* *no*
  - modal O-corner: *possible*$_1$ *not* $\equiv$ *possible*$_2$ *or* *impossible*
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Different versions of the quantifier square

- since the O-corner is not primitively lexicalized, it needs to be expressed in terms of one of the other corners
- in the literature we find at least two versions of the square

\[
\begin{array}{cccc}
\text{all } S \text{ are } P & \text{no } S \text{ are } P & \text{all } S \text{ are } P & \text{no } S \text{ are } P \\
\text{some } S \text{ are } P & \text{some } S \text{ are } \text{not } P & \text{some } S \text{ are } P & \text{not all } S \text{ are } P \\
\end{array}
\]
From the perspective of duality

- \textit{some} \(S\) are not \(P\) = \text{INEG}(\textit{some} \(S\) are \(P\))
- \textit{not all} \(S\) are \(P\) = \text{ENEQ}(\textit{all} \(S\) are \(P\))
A third version of the quantifier square

- \( O = \text{INEG}(I) \) and \( O = \text{ENEG}(A) \), but also \( O = \text{DUAL}(E) \)

- \textbf{not no} S are \textbf{not} P

- cognitive processing difficulties
- the O-corner is itself not primitively lexicalized
- but it can be non-primitively expressed in three ways, viz. as a duality-theoretic variant of each of the three other corners

```
all   no   all   no   all   no
some  INEG(some)  some  ENEG(all)  some  DUAL(no)
```
How about the A-corner?

- A is primitively lexicalized as *all*

- \( A = \text{INEG}(E) \)

- E is primitively lexicalized as *no*

- so A is non-primitively lexicalized as *no not*

- \( A = \text{DUAL}(I) \)

- I is primitively lexicalized as *some*

- so A is non-primitively lexicalized as *not some not*

- \( A = \text{ENEG}(O) \)

- O is itself not primitively lexicalized

- so A gets no additional non-primitive lexicalization
<table>
<thead>
<tr>
<th>Corner</th>
<th>INEG</th>
<th>ENEG</th>
<th>DUAL</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>O-corner</td>
<td>\textit{some}</td>
<td>\textit{all}</td>
<td>\textit{no}</td>
<td>\textit{some} not</td>
</tr>
<tr>
<td>A-corner</td>
<td>\textit{no}</td>
<td>\textit{all}</td>
<td>\textit{some}</td>
<td>\textit{all}</td>
</tr>
<tr>
<td>I-corner</td>
<td>\textit{no}</td>
<td>\textit{all}</td>
<td>\textit{some}</td>
<td>\textit{some}</td>
</tr>
<tr>
<td>E-corner</td>
<td>\textit{all}</td>
<td>\textit{some}</td>
<td>\textit{no}</td>
<td>\textit{no}</td>
</tr>
</tbody>
</table>
Summary for the quantifier square

A
DUAL(I)
INEG(E)

INEG(A)
ENEG(I)
E

DUAL(A)
I
ENEG(E)

ENEG(A)
DUAL(E)

all S are P
not some S are not P
no S are not P

all S are not P
not some S are P
no S are P

not all S are not P
some S are P
not no S are P

not all S are P
some S are not P
not no S are not P
Analogous story for the modal square

Duality and Lexicalization in the Square – L. Demey
interaction between duality and lexicalization

- the square has 4 corners (Klein 4-group)
- each corner has only 3 primitive formulations (lexicalization constraint)

the A-, I- and E-corner

- primitive lexicalization
- duality-theoretic variants of the **two** other primitively lexicalized corners

the O-corner

- no primitive lexicalization
- duality-theoretic variants of the **three** other corners

lexicalization has effects on **all** corners of the square (not just O)
A thought experiment

- what if O did have a primitive lexicalization, e.g. *nall*?
- each of the four corners would have four equivalent formulations:
  - one primitive lexicalization
  - duality-theoretic variants of the three other corners
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Overview

- individual authors:
  - Peter Abelard 1079 – 1142
  - William of Sherwood 1205 – 1270
  - Peter of Spain 1205 – 1277
  - Thomas Aquinas 1225 – 1274
  - William of Ockham 1287 – 1347
  - John Buridan 1300 – 1360
  - John Wyclif 1330 – 1384
  - Antoine Arnauld & Pierre Nicole (Port-Royal) 1662
  - Jacques Maritain (neo-Thomism) 1882 – 1973

- special topic of interest: mnemonic words for the square’s four corners
- mnemonic verses for the equipollences
- mnemonic verses for the Aristotelian/duality interplay
Peter Abelard (ca. 1079 – 1142)
Peter Abelard on the modal square

- *Dialectica* (ed. L. M. de Rijk, 1956)
- discussion of modal logic (singular propositions)
- four *ordines propositionum*:

  - *equivalence*: *Sunt enim omnes cuiuslibet ordinis propositiones ad se aequipollentes*
  
  - *contradiction*: *Et sunt quidem propositiones secundi dividentes cum propositionibus primi, et quarti cum tertii*
  
  - *subalternation*: *Inferunt autem propositiones quarti propositiones primi, sed non convertitur; et propositiones secundi propositiones tertii, sed non convertitur*
Abelard had all the ingredients for the **purely modal** square (i.e. singular propositions, no quantifiers):

- the four sets of three **equivalent propositions**
- the Aristotelian **relations** between (the propositions in) those sets
- the square as an actual two-dimensional **diagram**
  - square for the quantifiers in *Glossae super Peri Hermeneias*
  - square for the ‘binary’ quantifiers (*both, neither*, etc.) in the *Dialectica*

However, as far as we know, he never drew the modal square with three equivalent propositions per corner.

Abelard tried to extend his system to **quantified modal propositions**, but those attempts are “rather confused” (Lagerlund 2000).

Abelard’s **quantifier square** cannot have three propositions per corner:

- e.g. *some not* and *not all* are not logically equivalent for Abelard
- the former has existential import, the latter does not
Peter of Spain on the quantifier square

- *Summulae Logicales* (ed. L. M. de Rijk, 1973): quantifier square with one proposition per corner
• *si alicui signo preponatur negatio*, equipollet suo *contradictorio*

• Peter’s (only) example:
  - *non omnis* homo *currit*
  - *quidam* homo *non currit*

• *si alicui signo universali postponatur negatio*, equipollet suo *contrario*

• one of Peter’s examples:
  - *omnis* homo *non est animal*
  - *nullus* homo *est animal*

• *si alicui signo universali vel particulari preponatur et postponatur negatio*, equipollet suo *subalterno*

• one of Peter’s examples:
  - *non omnis* homo *non currit*
  - *quidam* homo *currit*
combine:
- the quantifier square (with one proposition per corner)
- the rules for duality in the quantifier square

Peter had all the resources to draw a quantifier square with three equivalent propositions per corner

however, as far as we know, he never actually did so

in some manuscripts of the *Summulæ*, we find mnemonic versions of the rules as well as their results:
- *Prae contradic, post contra, prae postque subalter*
- *non omnis – quidam non; omnis non quasi nullus; non nullus – quidam; sed nullus non valet omnis; non aliquis – nullus; non quidam non valet omnis; (non alter – neuter; neuter non prestat uterque.)*
Mnemonic verses

- verse for the rule (*Prae contradic, post contra, prae postque subalter*)
  - also in William of Sherwood, *Introductiones in Logicam*
  - also in John Wyclif, *Tractatus de Logica*

- verse for the results: different (clearer!) version in Sherwood
  - *Equivalent omnis, nullus non, non aliquis non.*
    - *Nullus, non aliquis, omnis non equiparantur.*
    - *Quidam, non nullus, non omnis non sociantur.*
    - *Quidam non, non nullus non, non omnis adherent.*

- 12th and 13th century: “a veritable craze for versifying” (Paetow 1910)
- the *Summulae*’s “greater success may be due to the fact that it contains more and better mnemonic verses than William of Shyreswood’s work.” (Kneale and Kneale 1964)
recall the following rule from Peter:

\[ \text{si alicui signo } universali \text{ postponatur negatio, equipollet suo } \text{contrario} \]

one might claim that Peter has forgotten the analogous rule:

\[ \text{si alicui signo } particulari \text{ postponatur negatio, equipollet suo } \text{subcontrario} \]

given the non-lexicalization of the O-corner, the latter rule is trivial

first rule: useful information about Latin/English

- \( \text{INEG}(A) = omnis \ non = nullus = E \)
- \( \text{INEG}(A) = all \ not = no = E \)

second rule: trivial

- \( \text{INEG}(l) = quidam \ non = quidam \ non = O \)
- \( \text{INEG}(l) = some \ not = some \ not = O \)
Peter of Spain on the modal square

- Peter draws a modal square with **four** equivalent propositions per corner.
- No need to differentiate between the first two in each corner:
  - In terms of **possibile** and **contigens**
  - ‘contingens’ convertitur cum ‘possibili’
- Essentially: modal square with **three** equivalent propositions per corner

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(only singular modal propositions; no quantified modal propositions)
Mnemonic terms for the corners of the modal square

- *purpurea, amabimus, illiace, edentuli*
  - each word stands for a **corner** of the modal square (with its four equivalent propositions)
  - each syllable stands for a **modality** (cf. next slide)
  - each vowel stands for a **combination of negations** (cf. next slide)

- contrast with the more well-known *barbara, celarent, etc.*:
  - each word stands for a **syllogism** (three non-equivalent propositions)
  - each syllable stands for a **proposition** (premise/premise/conclusion)
  - each vowel stands for a **quantifier** (AEIO convention)

- popular throughout history:
  - Peter of Spain
  - William of Sherwood
  - (Pseudo-)Aquinas
  - Port-Royal Logic
  - Jacques Maritain and other neo-Thomists
Mnemonic terms for the corners of the modal square

- *purpurea, amabimus, iliary, edentuli*

- the **order** of the syllables is significant:
  - syllable 1 \(\sim\) a proposition containing *possibile*
  - syllable 2 \(\sim\) a proposition containing *contingens*
  - syllable 3 \(\sim\) a proposition containing *impossibile*
  - syllable 4 \(\sim\) a proposition containing *necesse*

- independent **convention** for the vowels:
  - A: no negations at all
  - E: negation after the modality
  - I: negation before the modality
  - U: negation before and after the modality

### Klein 4-group:

- **O**
- **INEG(O)**
- **ENEG(O)**
- **DUAL(O)**
Example from Maritain

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John Buridan (ca. 1300 – 1360)
• *Summulae de Dialectica* (trans. G. Klima, 2001): quantifier square with six propositions per corner
Buridan has a quantifier square with six propositions per corner.

Consider, for example, the A-corner:

- **Omnis** homo *currit*
- **Nullus** homo *non currit*
- **Non quidam** homo *non currit*
- **Uterque istorum currit**
- **Totus homo est animal**
- **Quilibet homo est animal**

The last three are only relevant from a broader linguistic perspective: demonstratives, ‘binary’ quantifiers, mass nouns, free choice.

Quantifier square with three equivalent propositions per corner!
Buridan/Dorp on the quantifier and the modal square

- *Compendium totius Logicae* = later summary of the *Summulae* (by John Dorp in 1499, so 150 years after Buridan’s death)

- the *Compendium* contains
  - a quantifier square with three equivalent propositions per corner
  - a modal square with three equivalent propositions per corner
Buridan’s octagon for quantified modal propositions

- first CLAW/DWMC symposium (Demey & Steinkrüger 2017):
  - Buridan’s octagon can be understood as capturing the interaction between a quantifier square and a modal square
  - Buridan himself was already well aware of this

\[
\text{octagon} = \text{quantifier square} \times \text{modal square}
\]

\[
\begin{align*}
\text{9 propositions per corner} & \times \text{3 propositions per corner} = 27 \\
\text{3 propositions per corner} & \times \text{3 propositions per corner} = 9
\end{align*}
\]

- first symposium: focus on \(9 = 3 \times 3\)
- today: why 3 to begin with?

Duality and Lexicalization in the Square – L. Demey
<table>
<thead>
<tr>
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<th>quantifier square with 3 equivalent propositions per corner</th>
<th>modal square with 3 equivalent propositions per corner</th>
</tr>
</thead>
<tbody>
<tr>
<td>Peter Abelard</td>
<td>no!</td>
<td>no, but can be constructed</td>
</tr>
<tr>
<td>Peter of Spain</td>
<td>no, but can be constructed</td>
<td>yes</td>
</tr>
<tr>
<td>John Buridan</td>
<td>yes</td>
<td>yes</td>
</tr>
</tbody>
</table>
Thank you!

More info: www.logicalgeometry.org