The Logical Geometry of Russell's Theory of Definite Descriptions

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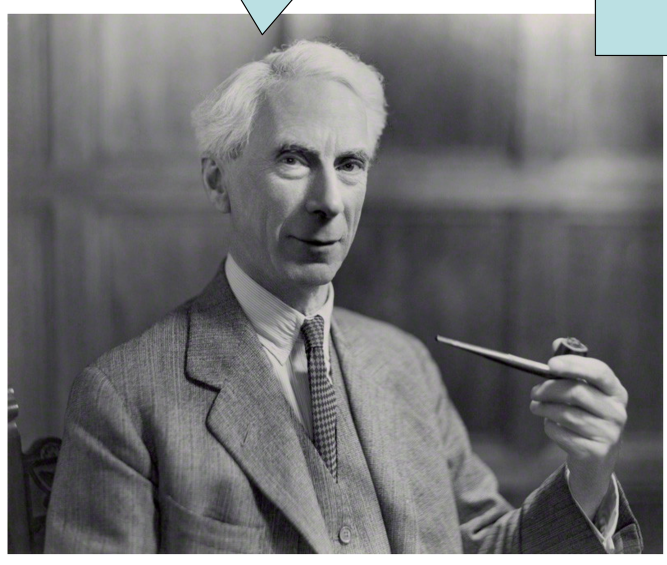
CONTEXT

"throughout modern times, practically every advance in science, in logic, or in philosophy has had to be made in the teeth of opposition from Aristotle's disciples"

On Denoting = "finest philosophical essay"



theory of definite descriptions = "paradigm of philosophy"

very influential in philosophy of language and formal semantics

contemporary study: **logical geometry**

first impression: **clear tensions!**

this paper

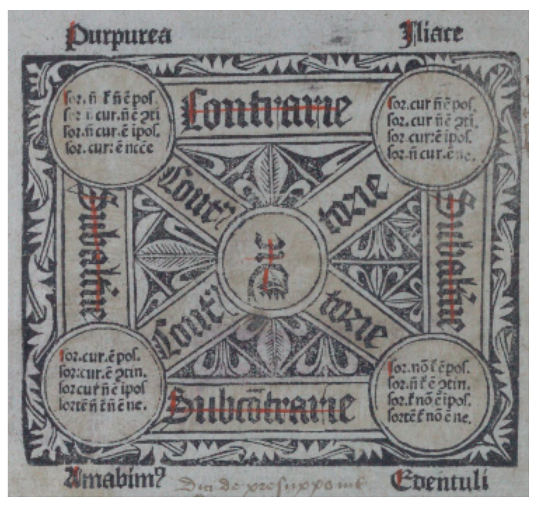
despite these apparent tensions:

fruitful interaction with advantages for both sides

syllogistics

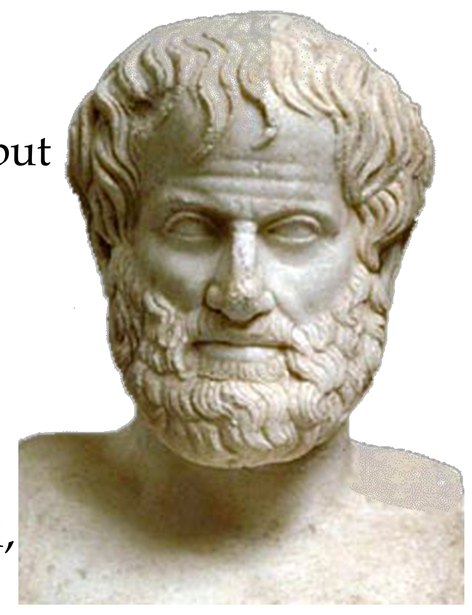
existential import

square of opposition



two propositions are

- *contradictory* iff they cannot be true together and they cannot be false together
- *contrary* iff they cannot be true together, but they can be false together
- *subcontrary* iff they can be true together, but they cannot be false together
- in *subalternation* iff the first one entails the second, but not vice versa



ARISTOTELIAN DIAGRAMS FOR DEFINITE DESCRIPTIONS

"the *A* is *B*" ⟹ $\exists x(Ax \wedge \forall y(Ay \to y = x) \wedge Bx)$

[the x: Ax]Bx

$$\begin{cases} (\text{EX}) & \exists x Ax \\ (\text{UN}) & \forall x \forall y((Ax \wedge Ay) \to x = y) \\ (\text{UV}) & \forall x(Ax \to Bx) \end{cases}$$

"the *A* is not *B*" — **ambiguous**

$\exists x(Ax \wedge \forall y(Ay \to y = x) \wedge \neg Bx)$

[the x: Ax]$\neg Bx$

versus

$\neg$[the x: Ax]Bx

$\neg \exists x(Ax \wedge \forall y(Ay \to y = x) \wedge Bx)$

natural addition: $\neg\exists x(Ax \wedge \forall y(Ay \to y = x) \wedge \neg Bx)$

$\neg$[the x: Ax]$\neg Bx$

☞ **weak** version of "the *A* is *B*": FOL-equivalent to $[(\text{EX}) \wedge (\text{UN})] \to$ [the x: Ax]Bx

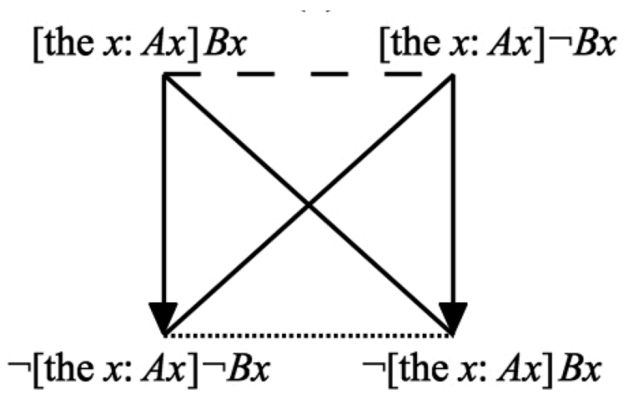
☞ importance for **self-predication principles**: $\neg$[the x: Ax]$\neg Ax$ is a FOL-tautology

the two interpretations of "the *A* is not *B*" stand in different Aristotelian relations to "the *A* is *B*":

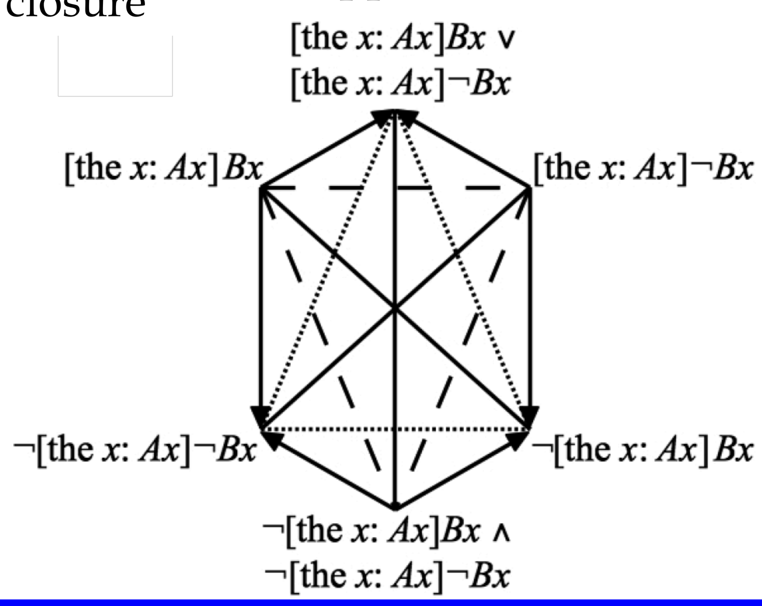
$\neg$[the x: Ax]Bx is FOL-**contradictory** to [the x: Ax]Bx

[the x: Ax]$\neg Bx$ is FOL-**contrary** to [the x: Ax]Bx

square of opposition — Boolean closure → **hexagon of opposition**



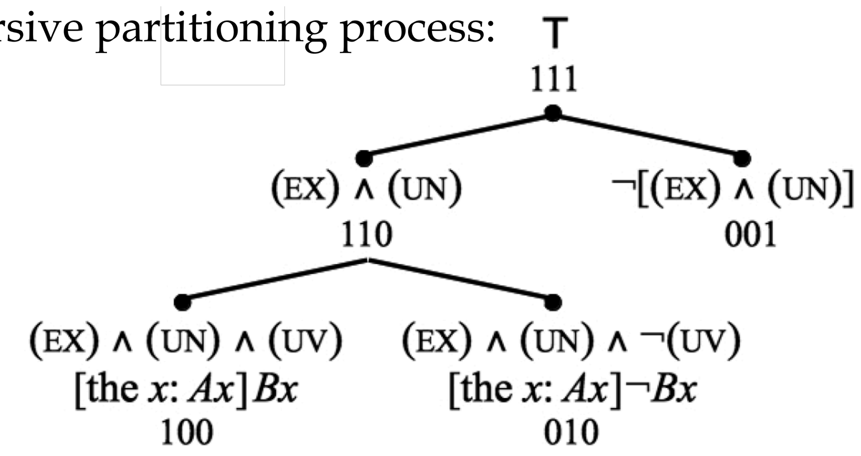
entirely in FOL
(no existential import)



bitstring semantics/partition of logical space induced by this square/hexagon:

$$\begin{Bmatrix} \alpha_1 & := & [\text{the } x\colon Ax]Bx, \\ \alpha_2 & := & [\text{the } x\colon Ax]\neg Bx, \\ \alpha_3 & := & \neg[(\text{EX}) \wedge (\text{UN})] \end{Bmatrix}$$

recursive partitioning process:



$\neg$[the x: Ax]Bx : negation of [the x: Ax]Bx relative to the **entire logical universe**

⇕ **semantic** characterization of the difference

[the x: Ax]$\neg Bx$: negation of [the x: Ax]Bx relative to the **restricted subuniverse**

DEFINITE DESCRIPTIONS AND THE CATEGORICAL STATEMENTS

"**all** *A* are *B*" $\forall x(Ax \to Bx)$

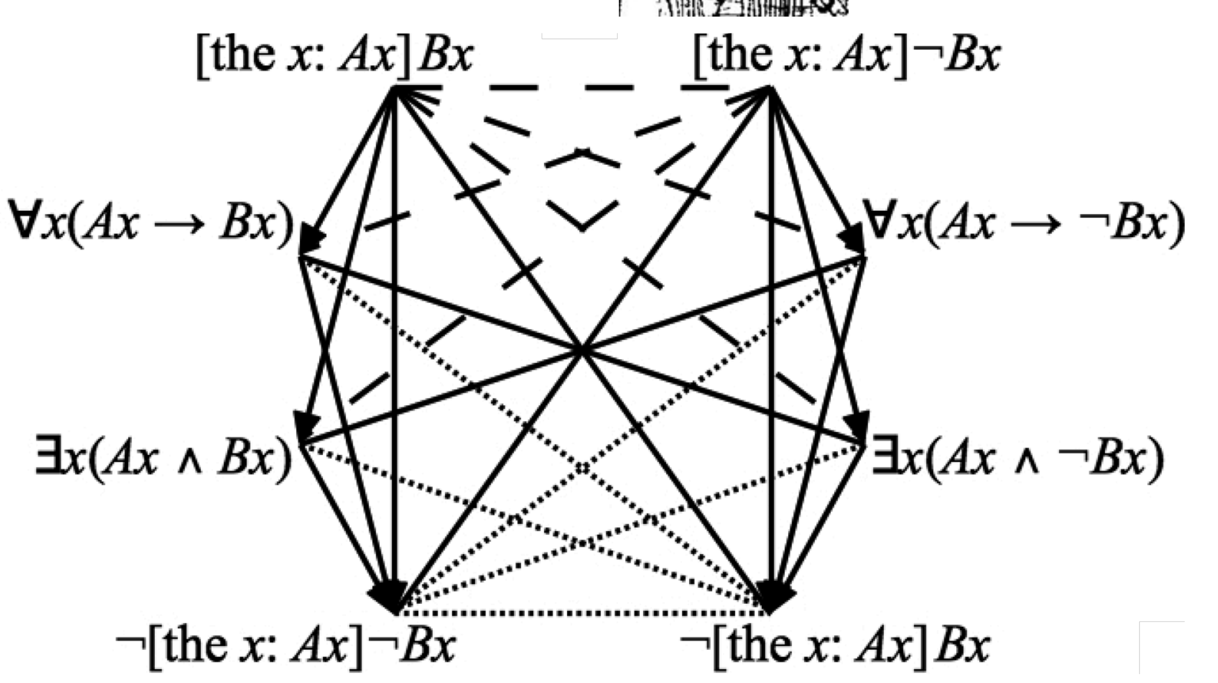
"**some** *A* are *B*" $\exists x(Ax \wedge Bx)$

"**no** *A* are *B*" $\forall x(Ax \to \neg Bx)$

"**some** *A* are **not** *B*" $\exists x(Ax \wedge \neg Bx)$

⇓

octagon of opposition similar to octagons found in **John Buridan** (14th C.)

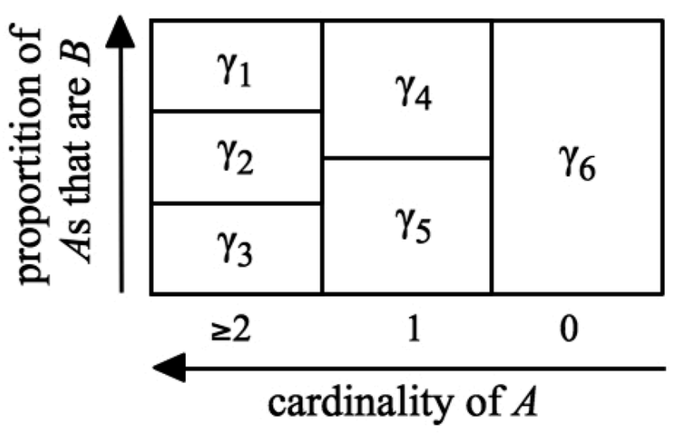


entirely in FOL
(no existential import)

bitstring semantics/partition of logical space induced by this octagon:

$$\left\{ \begin{array}{lll} \gamma_1 & := & \exists x \exists y(Ax \wedge Ay \wedge x \neq y) \wedge \forall x(Ax \to Bx), \\ \gamma_2 & := & \exists x(Ax \wedge Bx) \wedge \exists x(Ax \wedge \neg Bx), \\ \gamma_3 & := & \exists x \exists y(Ax \wedge Ay \wedge x \neq y) \wedge \forall x(Ax \to \neg Bx), \\ \gamma_4 & := & [\text{the } x\colon Ax]Bx, \\ \gamma_5 & := & [\text{the } x\colon Ax]\neg Bx, \\ \gamma_6 & := & \neg\exists x Ax \end{array} \right\}$$

ordered along two **semi-independent dimensions**:



other topics addressed in the **full paper**:

what happens if we move from FOL to syllogistics (i.e. assume that (EX) is a tautology)?

dually, what happens if we assume that (UN) is a tautoloy?

unexpected connection with another logical system: Public Announcement Logic

topics for **future research**:

what is a plausible recursive partitioning sequence for the octagon-induced partition?

more fine-grained version of this partition, by splitting up the "≥ 2" region into ">3" and "2" (cf. the words "both" and "neither" in English)

further connections with other logical systems

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