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Interactively Illustrating the Context-Sensitivity of Aristotelian Diagrams

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1 Introduction

2 Aristotelian Diagrams and Context-Sensitivity

3 Measuring Context-Sensitivity of Aristotelian Diagrams

4 Case Study: Categorical Statements with Subject Negation

5 Conclusion

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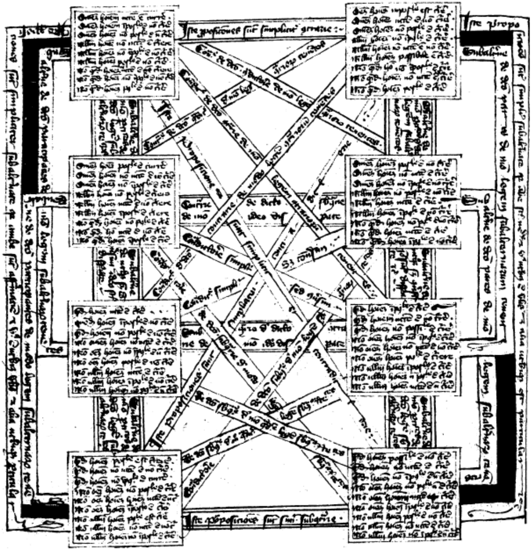
- Aristotelian diagram
  - compact visual representation
  - of the elements of some logical/lexical/conceptual field
  - and the logical relations holding between them

- most widely known example: square of oppositions

- intellectual background
  - rich history in philosophical logic
    - starting in the 2nd century AD (Apuleius)
    - especially popular in medieval logic
  - today: used in various disciplines
    - cognitive science, linguistics, law...
    - computer science, neuroscience...

⇒ Aristotelian diagrams as a *lingua franca* for an interdisciplinary research community concerned with logical reasoning


Purpurea
Iliace
Contrarie
Subalterne
Subalterne
Subcontrarie
Amabimus
Eventuli
Incipit Secundus Tractatus



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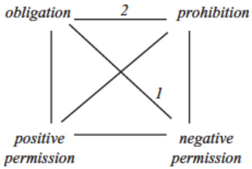
The Definition of 'Norm Conflict' in International Law and Legal Theory

Erich Vranes*

The possible set of inter-relations can be illustrated by using the so-called deontic square, which in fact relies on the logic square known since Greek antiquity,[85] and which was arguably first used in deontic logic by Bentham:[86]



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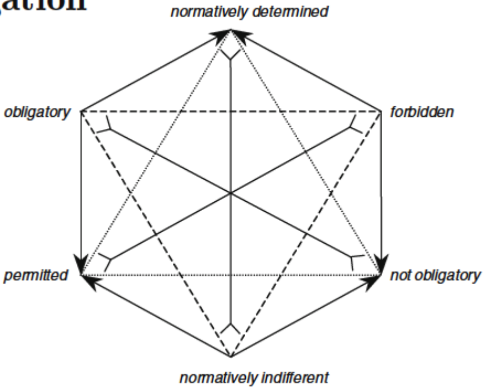
Deontological Square, Hexagon, and Decagon: A Deontic Framework for Supererogation

Jan C. Joerden



- research project: logical geometry (with Hans Smessaert)
- study **new decorations** of Aristotelian diagrams
  - historical case studies (e.g. Avicenna, Sherwood, Ockham)
  - applications in various fields (e.g. philosophy of language, AI)
- study Aristotelian diagrams as **objects of independent interest**
  - visual-geometrical aspects: dimension, perpendicularity, collinearity, etc.
  - abstract-logical aspects: information, graded opposition, etc.

$\Rightarrow$ logical context-sensitivity of Aristotelian diagrams!

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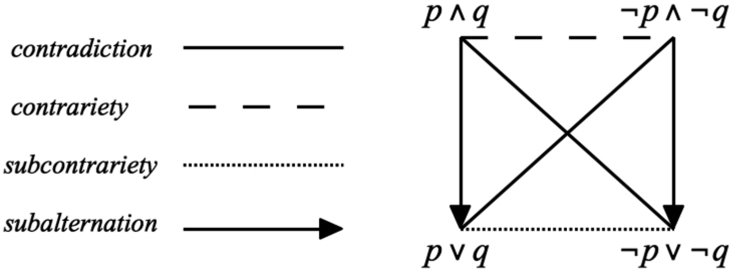
- traditional definition: two formulas are

contradictory	iff	they cannot be true together and they cannot be false together,
contrary	iff	they cannot be true together but they can be false together,
subcontrary	iff	they cannot be false together but they can be true together,
in **subalternation**	iff	the first entails the second but not vice versa.


contradiction
contrariety
subcontrariety
subalternation
$p \wedge q$
$\neg p \wedge \neg q$
$p \vee q$
$\neg p \vee \neg q$

- traditional definition: two formulas are

contradictory	iff	they cannot be true together and they cannot be false together,
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- where is the logical context-sensitivity?
  - in the modals ("can", "cannot")
  - e.g. "φ and ψ can be true together" $\rightsquigarrow$ "$\varphi \wedge \psi$ has a **model**"

- for a given logical system S, the formulas φ and ψ are

S-**contradictory**	iff	$S \models \neg(\varphi \wedge \psi)$	and	$S \models \neg(\neg\varphi \wedge \neg\psi)$
S-**contrary**	iff	$S \models \neg(\varphi \wedge \psi)$	and	$S \not\models \neg(\neg\varphi \wedge \neg\psi)$
S-**subcontrary**	iff	$S \not\models \neg(\varphi \wedge \psi)$	and	$S \models \neg(\neg\varphi \wedge \neg\psi)$
in S-**subalternation**	iff	$S \models \varphi \rightarrow \psi$	and	$S \not\models \psi \rightarrow \varphi$

- example from epistemic logic: formulas Kp and $\neg KKp$
  - contradictory in the system S4
  - subcontrary in the system T
  - only difference between these two systems: positive introspection axiom ($K\varphi \rightarrow KK\varphi$)

- philosophical importantance
  - logical system = list of axioms
  - but also: reflection of substantial position in philosophical debate (e.g. in epistemology: internalism vs. externalism)

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- how can we measure/quantify this type of context-sensitivity?

- bitstring representation of formulas
  - for a given logic S and fragment $\mathcal{F}$ of formulas, define the partition $\Pi_{\mathsf{S}}(\mathcal{F}) := \{\bigwedge_{\varphi\in\mathcal{F}} \pm\varphi\} - \{\bot\}$
    - mutually exclusive: $\mathsf{S} \models \neg(\alpha_i \wedge \alpha_j)$ for distinct $\alpha_i, \alpha_j \in \Pi_{\mathsf{S}}(\mathcal{F})$
    - jointly exhaustive: $\mathsf{S} \models \bigvee \Pi_{\mathsf{S}}(\mathcal{F})$
  - theorem: every $\varphi \in \mathcal{F}$ is S-equivalent to a disjunction of $\Pi_{\mathsf{S}}(\mathcal{F})$-formulas (relativized disjunctive normal form)
  - if $\Pi_{\mathsf{S}}(\mathcal{F}) = \{\alpha_1, \alpha_2, \alpha_3, \alpha_4, \alpha_5\}$, and $\varphi \equiv_{\mathsf{S}} \alpha_2 \vee \alpha_3 \vee \alpha_5$, then represent φ as the bitstring 01101

- we'll be interested in the following quantities:
  - $|\mathcal{F}|$: **fragment size**
  - $|\Pi_{\mathsf{S}}(\mathcal{F})|$: partition size, i.e. **bitstring length**

- relation between fragment size (n) and bitstring length (ℓ)
  - n-range: $R_n := \{\ell \in \mathbb{N} \mid \lceil \log_2(n+2) \rceil \leq \ell \leq 2^{\frac{n}{2}}\}$
  - theorem: for all $\ell \in R_n$, there exists a fragment $\mathcal{F}$ of size n and there exists a logical system S such that $|\Pi_{\mathsf{S}}(\mathcal{F})| = \ell$

    (note: both the 'fragment parameter' ($\mathcal{F}$) and the 'logic parameter' (S) are allowed to vary, i.e. are being quantified over)

- proposal: the logical context-sensitivity of a given fragment $\mathcal{F}$ with respect to a given set $\mathcal{S}$ of logical systems is positively correlated with the number of values in the $|\mathcal{F}|$-range that are reached if
  - the 'fragment parameter' is fixed to $\mathcal{F}$
  - the 'logical system parameter' varies within $\mathcal{S}$

- $\mathcal{F}^\dagger = \{all(A,B), all(A,\neg B), some(A,B), some(A,\neg B)\}$
(the four usual categorical statements)
- $R_{|\mathcal{F}^\dagger|} = R_4 = \{\ell \in \mathbb{N} \mid \lceil \log_2(4+2) \rceil \leq \ell \leq 2^{\frac{4}{2}}\} = \{3,4\}$

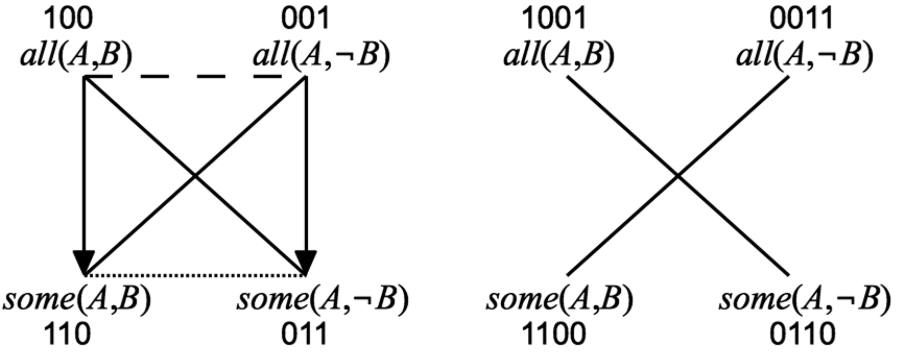
- $\mathcal{S}^\dagger$ contains just two logical systems:
  - FOL: first-order logic
  - SYL: classical syllogistics (= FOL + additional axiom $\exists x Ax$)

- one can show that $|\Pi_{\mathsf{FOL}}(\mathcal{F}^\dagger)| = 4$ and $|\Pi_{\mathsf{SYL}}(\mathcal{F}^\dagger)| = 3$

- by fixing the fragment parameter to $\mathcal{F}^\dagger$ and varying the logic parameter over $\mathcal{S}^\dagger$, $\frac{2}{2} = 100\%$ of the values in the $|\mathcal{F}^\dagger|$-range are reached

$\mathcal{F}^\dagger$ is maximally context-sensitive with respect to $\mathcal{S}^\dagger$

A Brief Illustration



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- extend the fragment and set of logics from the previous example:
  - $\mathcal{F}^\dagger \subset \mathcal{F}^\ddagger$ $\quad |\mathcal{F}^\dagger| = 4, |\mathcal{F}^\ddagger| = 8$
  - $\mathcal{S}^\dagger \subset \mathcal{S}^\ddagger$ $\quad |\mathcal{S}^\dagger| = 2, |\mathcal{S}^\ddagger| = 64$

- $\mathcal{F}^\ddagger$ contains the categorical statements with subject negation:

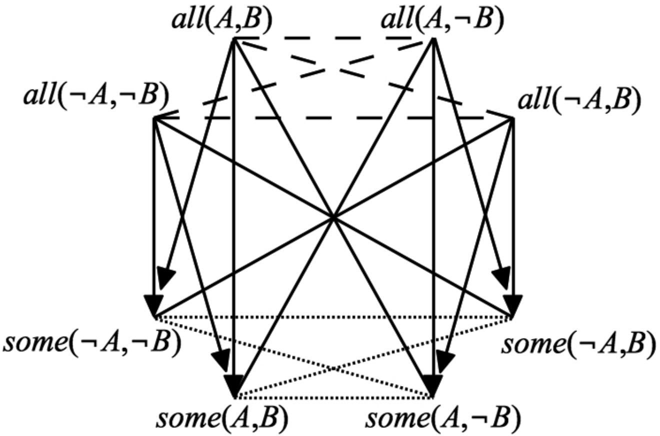
$$\begin{array}{llll} all(A,B) & all(A,\neg B) & some(A,B) & some(A,\neg B) \\ all(\neg A,B) & all(\neg A,\neg B) & some(\neg A,B) & some(\neg A,\neg B) \end{array}$$

- six axioms:

$$\begin{array}{llllll} A1 & \exists x Ax & A3 & \exists x Bx & A5 & \exists x \neg(Ax \leftrightarrow Bx) \\ A2 & \exists x \neg Ax & A4 & \exists x \neg Bx & A6 & \exists x \neg(Ax \leftrightarrow \neg Bx) \end{array}$$

- set of logics $\mathcal{S}^\ddagger := \{\mathsf{FOL} + \mathcal{A} \mid \mathcal{A} \subseteq \{A1, A2, A3, A4, A5, A6\}\}$

- Aristotelian octagon for $\mathcal{F}^{\ddagger}$ in the logic FOL $+ \{A1, A2, A3, A4\}$ (studied by Keynes & Johnson at the end of the 19th century)



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- interactive application to illustrate this
  - available online: logicalgeometry.org/octagon_context.html
  - heuristic role in ongoing research **DEMO!**
- some theoretical results
  - easy calculation: $R_8 = \{4, 5, \ldots, 15, 16\}$ $\longrightarrow$ 13 values
  - using application: fixing $\mathcal{F}^{\ddagger}$ and varying within $\mathcal{S}^{\ddagger}$ $\longrightarrow$ 8 values
  - "of all the bitstring lengths that might theoretically be necessary to represent an arbitrary 8-formula fragment with respect to an arbitrary logical system, about $\frac{8}{13} = 62\%$ is already necessary to represent the specific fragment $\mathcal{F}^{\ddagger}$ with respect to the specific logics in $\mathcal{S}^{\ddagger}$"
  - highest value that is reached: $|\Pi_{\mathsf{FOL}+\emptyset}(\mathcal{F}^{\ddagger})| = 16$
  - lowest value that is reached: $|\Pi_{\mathsf{FOL}+\{A1,A2,A3,A4,A5,A6\}}(\mathcal{F}^{\ddagger})| = 5$
  - Keynes & Johnson: $|\Pi_{\mathsf{FOL}+\{A1,A2,A3,A4\}}(\mathcal{F}^{\ddagger})| = 7$
  - inverse correlation between deductive strength and bitstring length (more axioms $\leftrightsquigarrow$ shorter bitstrings)

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Conclusion

- this talk:
  - explained logical context-sensitivity of Aristotelian diagrams
  - proposed a way to measure this context-sensitivity (bitstring lengths)
  - presented a case study: categorical statements with subject negation
  - illustrated it by means of an interactive application

- future work:
  - apply these results in historical analysis
    (e.g. Keynes/Johnson vs. Reichenbach)
  - investigate other sources of logical context-sensitivity
    in Aristotelian diagrams (e.g. contingency constraint)

Thank you!

More info: www.logicalgeometry.org